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'Playing the Game' of Story Problems: Situated Cognition in Algebra Problem Solving

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‘Playing the Game’ of Story Problems: Situated Cognition in Algebra Problem Solving

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Dedication

This work is dedicated to my father Robert DiBiano for giving me my love of mathematics, pushing me to pursue it, not allowing me to give up, and supporting and helping me every step of the way.

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'Playing the Game' of Story Problems: Situated Cognition in Algebra Problem Solving

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The importance of mathematics instruction including "real life" contexts relevant to students' lives and experiences is widely acknowledged (Common Core State Standards Initiative, 2010; National Council of Teachers of Mathematics, 2000; 2006; 2009), however questions about why contextualized mathematics is beneficial and how different types of contextualization impact problem solving have yet to be fully addressed by research. Common justifications for contextualized mathematics include the idea that relevant contexts may help students to apply what they learn in school to out-of-school situations, and that relevant contexts may scaffold learning by providing a bridge between what students understand and the content they are trying to learn.

The present study investigates these justifications, as well as students' beliefs and problem-solving methods, using story problems on linear functions. A situated cognition theoretical framework (Greeno, 2006) is used to interpret student behavior in the complex, social system of "school mathematics." In a series of interviews, students from a low-performing urban school were presented with algebra problems. Some problems were personalized to the ways in which they described using mathematics in their everyday lives, while others were normal story problems, story problems with equations, or abstract symbolic equations.

Results showed that students rarely explicitly used situational knowledge when solving story problems, had consistent issues with verbal interpretation of stories, and engaged in non-coordinative reasoning where they bypassed the intermediate step of understanding the given situation before trying to solve the problem. After completing

most of Algebra I, students still had considerable difficulty with symbolic representations, and struggled to coordinate formal and informal mathematical reasoning. Problems with the same mathematical structure with different amounts of verbal and symbolic support elicited different strategies from students, with personalized problems having high response rates and high use of informal strategies. This suggests that students can use sophisticated, situation-based reasoning on contextualized problems, and that different problem framings may scaffold learning. However, results also demonstrated that the culture of schooling, and story problems as an artifact of this culture, undermines many of the justifications for contextualizing mathematics, and that students need more authentic ways to develop their mathematical reasoning.

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I. Chapter One: Introduction

In the 2001 book *Radical Equations*, Moses and Cobb made a strong case for the importance of algebra as central to math literacy and economic vitality for today's students:

> Algebra was assigned a certain role, a certain place in the educational system. Students learned how to manipulate abstract symbolic representations for underlying mathematical concepts. Now here comes history, which brings in technology that places abstract symbolic representations front and center. These representations are tools to control technology. (p. 13)

Moses and Cobb argue that math literacy is essential to the struggle for equality and access for students from diverse backgrounds. More recently, Bob Moses and other activists began organizing a movement aimed to guarantee a quality education as a constitutional right. Moses (2010) writes:

> In the twenty-first century, we should pick our constitution up with the concept of a constitutional person thick enough to obligate the nation to secure for all its children a quality public school education as a matter of course, a matter of history, and a matter of our constitutional democracy. (p. 90)

However, defining a "quality" education in a practical and operational way can be a difficult task. At the Algebra Project 25th Anniversary Conference in 2008, led by Bob Moses, 150 activists, teachers, scholars, elders, and young people brainstormed what constitutes a quality education. A long list was created, with responses ranging from "empowers students to compete and excel in a global society" to "connects to a child's life story and circumstances" to "challenges, broadens, props up students' voices." While these are lofty and important goals, there is some consensus in mathematics education, in many of the publications to be reviewed here specifically, that these goals are not being met for many young people in public schools today.

Bob Moses identified the critical role algebra plays in the math sequence, functioning as a gatekeeper to higher mathematics and use of abstract mathematical tools. Research aiming to make algebra instruction more accessible to a wide range of students must be a priority. While there are a number of ideas that give promising directions for mathematics education, such as the Algebra Project's conception of "experiential learning," (Moses & Cobb, 2001) the "models and modeling perspective" (Lesh & Zawojewski, 2007), and the "functions-based" approach (Chazan, 1999), researchers cannot ignore the current status of algebra instruction in middle and high school, and the ways in which practices are disconnected from advances in the learning sciences. By understanding the limitations of current methods of curriculum, instruction, and assessment in algebra, the mathematics education community can make a stronger case for the need for change. This paper explores one facet of traditional mathematics instruction – story problems – and considers whether a system in which these scenarios represent situated practice is truly providing a quality education for all students.

A. Contextualization in Mathematics

The importance of mathematics instruction including "real life" contexts relevant to students' lives and experiences is widely acknowledged (Common Core State Standards Initiative, 2010; National Council of Teachers of Mathematics, 2000; 2006; 2009), including internationally (Palm, 2009; Xin, 2009). The recently released Common Core State Standards (CCSS) describe eight standards for mathematical practice, which include modeling with mathematics: "Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace" (Common Core State Standards Initiative, 2010, p. 2). Similarly, recent principles and standards (NCTM, 2000), curriculum focal points (NCTM, 2006), and a conceptual framework for curriculum and instruction (NCTM, 2009) published by the National Council of Teachers of Mathematics accentuate the importance of teaching mathematics in context:

> Mathematics should help students understand and operate in the physical and social worlds. They should be able to connect mathematics with a real-world situation through the use of mathematical models. The connections between mathematics and real-world problems developed in mathematical modeling add

> value to, and provide incentive and context for, studying mathematical topics. (NCTM, 2009, p. 2)

Despite this attention, questions about why contextualized mathematics is beneficial and how different types of contextualization impact student beliefs and problem-solving behaviors have yet to be fully addressed by research.

The primary way in which mathematics is contextualized in many classrooms today is through story problems or word problems (Jonassen, 2003). Here the primary focus is on "traditional story problems," which are defined as relatively short and closed-ended problems embedded in contexts that reference objects, people, and events from the world (i.e. "real world" contexts), and that have a specific, pre-determined answer that can be arrived at by using the given information. Story problems have received much attention in the mathematics education literature, and this attention is warranted given that story problems permeate mathematics curricula from kindergarten to undergraduate courses, and there is little evidence that this trend is changing (Jonassen, 2003). Nearly half of the problems on the 2009 ninth grade standardized mathematics assessment for Texas are traditional story problems (TEA, 2009), and a recent national survey of 743 Algebra I teachers compiled for the National Mathematics Advisory Panel showed that "solving word problems" was considered the most serious deficiency of incoming algebra students (Loveless, Fennel, Williams, Ball, & Banfield, 2008).

Beyond just their prevalence in schooling, studying how students solve story problems provides insight into how situational contexts that are sometimes thought or intended to represent applied problem solving interact with formal mathematical ideas. In an educational climate where focus has been on making mathematics more meaningful to a diverse population of students (i.e. NCTM, 2000; 2009), story problems have become a common way to accomplish this goal in traditional mathematics instruction.

B. Justifications for Contextualization: Symbol and Verbal Precedence

In the case of algebra, two views of the relationship between story problems and mathematics formalism (viewed by some as mathematics content) are prevalent today. In the *symbol precedence view*, algebraic symbolism should be presented first, and story problems are then used as a way to apply these formalisms. A competing position is the *verbal precedence view*; from this perspective, verbal skills develop before symbol manipulation skills, and thus instruction on verbal problems like story problems should be presented before symbolic equations (Nathan & Petrosino, 2003).

From a symbol precedence perspective, the primary purpose of story problems is to solve the "transfer problem"; by giving students contextualized problems in addition to abstract problems, they will be better prepared to face the demands of using mathematics in everyday situations and in the workplace. From a verbal precedence perspective, story problems have a different purpose - context can provide accessibility or scaffolding for students, with concrete and familiar experiences providing a bridge between what the students know and the abstract mathematics they are trying to learn (Boaler, 1994). Studies have shown that while many teachers and textbooks subscribe to symbol precedence views, student performance better corresponds to a verbal precedence model (Koedinger & Nathan, 2004; Nathan & Koedinger, 2000a; 2000b; Nathan, Long, & Alibali, 2002; Nathan & Petrosino, 2003).

Research on arithmetic story problems has called into question whether the common justifications behind either of these two models are complete given the situated nature of problem solving. In the situated cognition perspective discussed by Greeno (2006), intelligent behavior takes place in complex social systems that include learners, teachers, curriculum materials, and the physical environment, as well as representational, material, and conceptual resources. Under this framework, "school mathematics" represents its own social system whose norms, standards, and practices are different from problem solving in other contexts, including how mathematics is used in everyday contexts and in the workplace. This disconnect has been evidenced by research showing how problem solving in school mathematics differs from applied problem solving of professionals and practitioners (Lave & Wenger, 1991; Masingila, Davidenko, & Prus-Wisniowska, 1996; Resnick, 1987; Saxe, 1988; Taylor, 2005), by research suggesting that students rarely apply everyday knowledge to stereotyped, oversimplified school-based tasks like story problems (Baranes, Perry, & Stigler, 1989; Greer, 1997; Palm, 2008; Reusser & Stebler, 1997; Xin, 2009), and by research demonstrating that such application can actually result in incorrect answers (Boaler, 1994; Cooper & Harries, 2009; Inoue, 2005; Kazemi, 2002; Roth, 1996).

C. The Authenticity of Story Problems on the Texas Standardized Assessment

To frame the arguments in this paper, four story problems from the 2006 ninth grade standardized mathematics assessment in Texas (TEA, 2006) will be discussed. Drawn upon for this discussion is Palm's (2008) definition for *authenticity* in word problems. Palm writes, "For a task with an out-of-school context to be authentic it must represent some task situation in real life, and important aspects of that situation must be simulated to some reasonable degree" (p. 40). Figure 1 shows a framework for word problem authenticity developed in Palm (2006); he describes these elements as "the aspects of real-life situations considered to be important in their simulation" (p. 44).

A. Event	F. Circumstances
B. Question	F1. Availability of external tools
C. Information/data C1. Existence C2. Realism C3. Specificity	F2. Guidance F3. Consultation and collaboration F4. Discussion opportunities F5. Time
D. Presentation D1. Mode D2. Language	F6. Consequences G. Solution requirements H. Purpose
E. Solution strategies E1. Availability E2. Experienced plausibility	H1. Purpose in the figurative context H2. Purpose in the social context

Figure 1. Framework from Palm (2006) showing the aspects of real situations that are important in simulated mathematical situations like story problems. Reproduced with permission of author.

The first two problems to be discussed using Palm's (2006) framework are shown in Figure 2. Considering problem 8 in Figure 2 in terms of Palm's definition authenticity, the process that Olga is using to measure the distance between two locations could be questioned. Certainly it makes the most sense that if this were a real situation, Olga would want to drive from one city to another, meaning that a direct measurement of the distance between the two cities on a map would be of limited use. According to Palm's (2006) framework, this problem has issues with *existence* – information that would be available in the real task, namely a visual depiction of the actual roads going between the cities, is missing in the simulated situation.

8 Olga plans to take a trip from her house in San Marcos, Texas, to a friend's house in Zapata, Texas. She measured the distance between the two places on a map and found it to be 8 inches. If the scale on the map is $\frac{1}{2}$ inch represents 14 miles, which is closest to the actual distance in miles between the two places?

F 112 mi

G 224 mi

H 56 mi

J 44 mi

45 A jar contains 6 red marbles and 10 blue marbles, all of equal size. If Dominic were to randomly select 1 marble without replacement and then select another marble from the jar, what would be the probability of selecting 2 red marbles from the jar?

A $\frac{9}{64}$

B $\frac{1}{8}$

C $\frac{3}{5}$

D $\frac{3}{8}$

Figure 2. Problems 8 and 45 from 2006 Released 9th grade TAKS Test
http://www.tea.state.tx.us/index3.aspx?id=3839&menu_id3=793

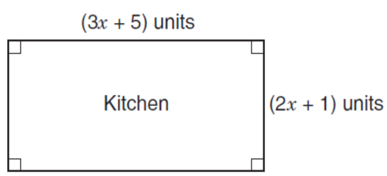
This is tied to issues with the *purpose* of solving the problem, another dimension of authenticity. If students perceive the purpose to be that Olga wants to figure out how much distance she will have to drive to get to her friend's house, a reasonable assumption given students' experiences with travel, they will be left without the information needed to make a judgment about a solution. To solve this problem, students must share the problem author's "common sense," meaning they must disregard what the most plausible purpose for making this calculation would be, and suspend what they know about actually traveling between two cities on roads. It is also worth mentioning that the days of children like Olga using a ruler to scale distances on a paper map, if there ever were such days, are over. Today's youth are adept at using programs like MapQuest, which calculates distances automatically, and thus they may be completely unfamiliar with the process that Olga is applying in terms of their personal experiences. This may lead to issues with what Palm (2006) refers to as *experienced plausibility*, or the match of strategies that would be applied in the real situation versus the simulated situation.

Figure 2 also shows problem 45 from the Texas state standardized test, where a character Dominic is selecting marbles from a jar. Palm (2006) gives such marble problems special attention, stating that "Picking marbles from an urn and noting their colours (a common event in probability word problems) is not something that people do in out-of-school life and therefore does not have a corresponding real event" (p. 44). If a problem fails along this most basic dimension of *event* authenticity, the other dimensions of authenticity become trivial to examine.

The problems in Figure 3 are of a different variety because they introduce an interaction between a story representation and a symbolic representation. Problem 16 in Figure 3 is puzzling from an authenticity standpoint. Why would Tammy use seemingly random algebraic expressions with constant rates of change to describe the length and width of her kitchen? Why would Tammy use variables when drawing her floor plan, rather than actually measuring her kitchen floor and putting in the real measurements? What are these "units" that Tammy is talking about? If she was really interested in drawing a floor plan, would it not be important to note what the units were? Why is Tammy making a floor plan anyways? Does she want to buy tiling? Is she trying to sell her house?

The inclusion of the symbolic representations in this problem leads to issues with *event authenticity*; although people do make floor plans of their kitchens, they do not make floor plans using linear functions. Further, there are clearly issues with *existence*; in the real situation Tammy would be able to obtain precise information about the measurements of her floor. This problem is also unclear with respect to *purpose*; the reader does not know why Tammy would want to find the area of her floor in the first place, much less use variables to do so. These authenticity issues raise an important question – why did the test developers decide to make this problem a story at all? Why did they not just show a rectangle with two variable dimensions? Does the introduction of this ficticous character Tammy and her strategy for measuring her kitchen really do anything other than make the question nonsensical?

16 Tammy drew a floor plan for her kitchen, as shown below.



Which expression represents the area of Tammy's kitchen floor in square units?

F $6x^2 + 30x + 5$

G $6x^2 + 13x + 5$

H $10x + 12$

J $5x + 6$

25 In many parades, flowers are used to decorate the floats. The table below shows the number of flowers used in each row of a parade float.

Row Number, r	Number of Flowers, n
1	54
2	58
3	62
4	66

Which equation best describes these data?

A $n = 2r + 52$

B $n = r + 54$

C $n = 4r + 50$

D $n = 4r + 54$

Figure 3. Problems 16 and 25 from 2006 Released 9th grade TAKS Test
http://www.tea.state.tx.us/index3.aspx?id=3839&menu_id3=793

Analyzing problem 25 in Figure 3, several issues also emerge. First, it is unclear what the "rows" being referred to are, or how they are significant to the problem. Are these rows of flowers in a design? Rows of benches being decorated? This may correspond to Palm's (2006) *language use* dimension of authenticity. The problem also has issues with *question authenticity* – the question posed here, finding a symbolic equation that relates row number to flowers, is not a question that would be asked in the real situation. In the actual situation, the question being asked would likely be something much more practical, like how many flowers need to be purchased for the float? This task also has issues with *purpose* – the reader does not know why finding the number of flowers corresponding to each row is important.

However, pushing beyond Palm's framework into the realm of algebra, there are two further issues with problems 16 and 25 that are important to bring to the forefront. First, these problem employ symbolic representations in a way they would not be used, for a purpose that they would have no ultility for. Unless Tammy's kitchen is going to grow, or she wants to build some sort of general kitchen dimensional model for future use, there is no legitamite reason for her to employ symbolism. Accordingly, it is difficult to imagine the float designers having any sort of realistic reason to want to model a count of flowers as it varies by row using algebraic expressions.

Second, both of these problems include functions that contain a constant rate of change – a foundational concept in Algebra I. However, in neither situation does a constant rate of change make sense or have any kind of realistic interpretation. In problem 25, why would each row of the parade float be decorated with exactly 4 more flowers than the previous row? The ways in which symbolic representations and rate of change are used in these problems are completely arbitrary, and this may lead to students viewing them as meaningless.

D. Discussion

The story problems presented in this section call into question the common justifications for placing mathematics in context. How can story problems provide accessibility if the contexts are not connected to actual experiences students may have? And how can story problems help students transfer mathematical learning to applied out-of-school problem solving if in fact the problems being posed bear little resemblence to the ways in which mathematics is used to solve real problems?

Traditional story problems may have a place in mathematics instruction, but research needs to address where precisely that place is by examining both their affordances and constraints as contextualized mathematics. The real issue is that because these types of problems are so prevalent in schooling today, especially on high-stakes assessments, a danger emerges of these being the only type of contextualized problems that students ever see. A case needs to be made for a more balanced approach, empirically grounded in the problem solving of students in traditional settings where story problems prevail as contextualized mathematics.

The present series of studies uses a situated cognition perspective to take a critical look at the two justifications introduced here for using story problems – story problems as a way to apply school mathematics learning to out-of-school situations, and story problems as a way to use concrete experiences to bridge abstract mathematical ideas. These studies investigate whether findings from situated studies of arithmetic story problems are applicable to algebra learning, and look at new considerations that arise through the interaction of story contexts with symbolic representations. This research critically examines students' arithmetic and algebraic ways of solving story problems on linear functions, and explores how problem framing (i.e. the characteristics of a problem's "cover story") affects problem-solving behaviors.

II. Chapter Two: Literature Review

A. Theoretical Framework

In the situated cognition perspective discussed by Greeno (2006), intelligent behavior takes place in complex, social systems or *activity systems* like schools, communities, and places of work and business. The "context" of an activity system like school includes learners, teachers, curriculum materials, and the physical environment, as well as representational, material, informational, and conceptual resources. Greeno (1991) describes how knowledge is constructed by interacting with people and resources in the "environment" that constitutes domain knowledge. Knowing in a domain involves understanding how to recognize and find resources relevant to your purposes and learning when and how to use these resources productively. Knowing in a domain also involves being able to navigate this environment, understanding how elements of the environment interact, and being able to create mental models as reasoning tools that emulate the affordances and constraints of the domain.

This view of knowledge stands in contrast to the information-processing perspectives from cognitive science, where a domain is a hierarchy of facts and procedures, and knowledge is an individual's accumulation of this hierarchy. From a situated cognition perspective, an individual's cognitive knowledge structures are understood in relation to the activity and interactions of the entire system, with the system being the unit of analysis rather than the mental processes of an individual. However, analysis across multiple levels, including individual cognition, is not precluded:

> Analyses of thinking processes and information structures perceived and constructed by one or more of the individuals participating in a group can be conducted, as can analyses of the ways in which the activity in a system is supported and constrained by the institutional setting of which the activity system is a part. (Greeno, 2006, p. 84)

From a situated cognition perspective, learning is viewed as a trajectory of participation as a member of a social system, or the way in which participation in activity contributes to growth as a learner and future participation in other activity systems of value to the learner (Cobb & Bowers, 1999; Greeno, 1997; Greeno, 2006). A situated cognition approach acknowledges that school itself is a complex, social system with its own norms, procedures, practices, and characteristics of performance, while also recognizing that in order for students to be successful, they need to engage in activities that encourage participation in other systems. From this perspective, communities and groups have the power to shape what counts as mathematics, including the meaning of terminology, concepts, and principles, and how they can be applied in practice by community members (Greeno, 2006).

This theoretical framework is a useful lens for understanding and interpreting students' problem-solving behaviors in school mathematics tasks such as story problems. A situated cognition perspective recognizes that school-based learning consists of participation structures specific to the school setting, which may not be especially useful or appropriate in other settings. For example, based on their experiences in school, students my come to believe that participation in mathematical activity is constituted by obtaining answers to short, self-contained problems requiring repetitive calculations, with no larger goal or purpose for which the answers have meaning. Although mathematics may be imbedded in "real world" contexts like riding a boat or calculating the cost of a cell phone plan, solving these problems in school may bear little resemblance to how people actually use mathematics as a tool in everyday and professional situations. Greeno (1991) frames this disconnect as being similar to trying to learn about a new place or environment using only maps and descriptions, rather than actually visiting the location. He cautions that this type of learning can be unrealistically limited to interactions with symbolic representations, rather than the first-hand experiences that are needed to function productively in the new environment.

Central to prior research on story problems has been how students activate "real world knowledge" or take into account "practical considerations" when problem solving. These analyses are meant to describe, for instance, whether the student takes into account what they practically know about jogging based on their everyday experiences with jogging when solving a story problem about jogging. However, from a situated cognition perspective, all knowledge is "real world," imbedded in different systems of participation, and all knowledge is "practical" with respect to the norms and expectations of the system the behavior is situated in. So in the analysis presented here, activation of real world knowledge is framed as instead being the degree to which students use the participation practices they use in everyday situations out of school when solving story problems in a school mathematics context. For brevity, this is sometimes referred to as "situational knowledge" or "everyday knowledge."

B. Situated Cognition Research in Mathematics

A number of important research studies focusing on situated views of primary and secondary mathematics classrooms have been conducted. Paul Cobb (Cobb & Bowers, 1999; Cobb, Stephan, McClain, & Gravemeijer, 2001; Cobb & Hodge, 2002) conducted design research in elementary and middle grade mathematics classrooms. This line of research coordinates an emphasis on individual meaning, or a psychological perspective, with a social perspective that takes into account the ways of "acting, reasoning, and arguing that are normative in a classroom community" (Cobb et al., 2001, p. 118). Cobb's work differentiates between general classroom social norms and so-called *sociomathematical norms*, which are aspects of classroom activity specific to the learning of mathematics. He further differentiates *mathematical practices* as being specific to a certain content area or idea in mathematics. Cobb's work accentuates how the norms of participation influence students' learning opportunities:

> In a very real sense, students who cannot participate in these practices are no longer members of the classroom community from a mathematical point of view. This situation is highly detrimental given that to learn is to participate and contribute to the evolution of communal practices. (Cobb & Bowers, 1999, pp. 9-10)

Related to this work is Cobb and Hodge's (2002) *relational perspective*, where diversity is conceptualized as students' participation in local and broader community practices outside of school, and equity is how the continuities and discontinuities between in-school and out-of-school participation practices affect access.

Other researchers have acknowledged a disconnect between school-based mathematics and mathematical practice. Lampert (1990) discusses how the *knowing* and *doing* of mathematics in traditional classrooms is inconsistent with how mathematics is practiced within the discipline. In the traditional classroom, mathematics is a static, certain body of knowledge passed down from authorities like teacher and text, and the primary practice of value is being able to obtain the right answer quickly, with this answer usually being kept implicit or private. This is contrasted with mathematical activity in the discipline, which involves making conjectures, forming explanations and assertions, questioning and challenging the thinking of others, and viewing reasoning and argument as the central authority in determining an idea's validity (Lampert, 1990).

Schoenfeld (1988) studied mathematics classes in a high-performing New York suburban district, focusing on high school geometry instruction. In these traditional classrooms, formal mathematical objects like geometric proofs were disconnected from problem solving and discovery. Students expected to be able to solve mathematics problems or "exercises" in short amounts of time, and accuracy and speed were valued. The *form* of the answer or mathematical expression, in this case the deductive proof, was of primary concern, rather than the underlying mathematics concepts. Students viewed themselves as passive recipients of mathematical procedures handed down from external authorities, and were expected to memorize and reproduce material with or without understanding. Schoenfeld alludes to the idea that students' beliefs about the nature of the discipline affect the knowledge they use or failed to use when solving problems, although this was not a central focus of his inquiry. He also makes it clear that the teacher's focus on mechanistic, rule-based procedures was reasonable, given that this knowledge was the focus of standardized exams.

Boaler (1998; 2002) conducted a 3-year ethnographic case study of two high schools in the United Kingdom. One school was using a traditional approach to mathematics instruction, while the other was using a project-based approach. Through this comparison, she revealed how students at the traditional school took a passive approach to their work and saw little reason to think about what they were doing. They viewed mathematics as consisting of rules, formulas and equations to be memorized, and when problems were posed that did not require obvious and simplistic application of a recently-covered procedure, students became confused and often stopped working. Students also utilized *cue-based* strategies where they would use nonmathematical prompts from the teacher or textbook to deduce aspects of the expected procedure and solution. Students' mathematical knowledge in the traditional setting was inert and procedural, and students largely did not see the connection between what they were learning in school and real situations where mathematics would be of use. This stood in stark contrast to the ways in which students in the project-based school used and understood mathematics, which was characterized by meaning-making, flexibility, and confidence.

Research on *The Adventures of Jasper Woodbury* video series also shows the importance of using mathematical procedures as tools in authentic situations in order to promote conceptual understanding (Cognition and Technology Group at Vanderbilt, 1990). The CTGV notes that, "We have shown that one of the advantages in learning in problem-solving contexts is that students acquire information about the conditions under which it is useful to know various concepts and facts" (p. 3). This can be contrasted with Boaler's discussion of the cue-based strategies that students at the traditional school used in the absence of an authentic problem-solving context from which to reason.

C. Research on Arithmetic Story Problems

Arithmetic story problems caught the attention of many researchers in math education following the results of the 1983 National Assessment of Educational Progress (NAEP). This assessment revealed that while U.S. students were able to solve routine, one-step story problems, they had difficulty with non-routine problems which required nonstandard approaches or an analysis of the story situation (Carpenter, Matthews, Lindquist, & Silver, 1983). Highlighted was a division story problem given to 13-year olds: “An army bus holds 36 soldiers. If 1,128 soldiers are being bussed to their training site, how many buses are needed?” (p. 491). Results showed that 29% of students included the remainder of the division problem in their response, even though it makes no sense in the context of the story, and another 18% ignored the remainder rather than including the additional needed bus. Based on the NAEP results, it was concluded that many U.S. students had not developed problem-solving skills and “attempt to apply mechanically some mathematical calculation to whatever numbers are given in a problem” (p. 490). As the remainder of the literature review will show, some 27 years since this assessment, there is little reason to believe the situation has changed.

Around the same period, research conducted on students solving arithmetic problems revealed that slight variations in problem wording often result in children using different types of strategies (Carpenter & Moser, 1984; Carpenter, Fennema, Franke, Levi, & Empson, 1999). Kintsch and Greeno (1985) developed a model of problem solving for arithmetic word problems where students first translate from a given problem to a *propositional textbase*, which is a conceptual representation of the relationships and concepts in the text. Students then form a *problem model* or *situation model* that infers the information needed to solve the problem based on students’ knowledge of the domain. Later research (Hegarty, Mayer, & Monk, 1995) recognized that unsuccessful problem solvers use *direct translation strategies*, operating on numbers and keywords from the problem text and bypassing an intermediate formation of a model of the situation. In contrast, successful problem solvers use *problem model strategies* where they form a mental representation of the situation and use this model to plan and assess their strategies.

Cummins, Kintsch, Reusser, and Weimer (1988) called attention to the issues that young children have with text comprehension in story problems, showing that students’ mistakes often represent correct answers to misinterpreted stories. Another important early study demonstrated that elementary students may or may not use their everyday knowledge when solving story problems, and that use of everyday knowledge depends on how the situational context interacts with the numbers given in the problem; for example, monetary units like 25 cents or time units like 15 minutes are easier to work with when contextualized (Baranes et al., 1989).

Recent international research on story problems has examined the strategies students use when given word problems that do not have enough information to be solved or that require “practical considerations” to be taken into account. The main finding is that students largely adhere to the norms of the mathematics classroom, making the assumption that all story problems follow a stereotyped pattern and have a direct computational answer based on the numbers given in the problem (Greer, 1997; Palm, 2008; Reusser & Stebler, 1997; Xin, 2009). An example of a “problematic” problem used in these studies is: “Martin’s best time to run 100 m is 10.00 sec. How long will it take him to run 10,000 m?” (Palm, 2008). What makes this story “problematic” is that the speed at which Martin could run 100 meters is not likely to be maintained if he runs 10,000 meters. However, many students in these studies do not attend to this distinction. Reusser and Stebler (1997) write, “As illustrated by data from our studies, most students perceived word problem solving as a puzzle-like activity with no grounding in factual real-world structures and with no relation to a goal-directed, more authentic activity of mathematization or realistic mathematical modeling” (p. 323).

However, other results show promise for students’ sense-making capabilities, and their ability to nullify prevailing sociomathematical norms if they believe the rules of the situation have changed. Wyndhamn and Saljo (1997) gave elementary students problematic story problems, but instead of working them individually on a quiz or test, the students worked the problems in collaborative groups. The study found that a strong majority of the student groups did decide to take situational considerations into account, and were able to conclude that the problems could not be solved. Palm (2008) found that when word problems were revised to better match his framework for authenticity (Figure 1), students’ use of everyday knowledge occurred in significantly higher proportions. Finally, Inoue (2009) found that although the majority of students in her study gave seemingly calculational answers to problematic word problems, they were able to give sensible experience-based rationales for these answers when prompted.

Masingila et al. (1996) describe a study they conducted which examined the mathematical problems that occur in the day-to-day work of carpet layers, a dietician, an interior designer, a retailer, and a restaurant manager. The researchers compared how each practitioner approached an applied problem in their field to the way high school students approached matched story scenarios. The study found that the two groups conceptualized their goals differently, with the students viewing the problem as a mathematical exercise to which learned procedures should be applied, and the practitioners viewing mathematics as a tool for solving the problem rather than as the goal of the problem. The study also revealed that the practitioners had a stronger understanding of the concepts behind each problem and were better able to take into account situation-based constraints. Taylor's (2005) dissertation on first and second grader's engagement in shopping practices and Saxe's (1988) study of Brazilian child candy sellers further show how the non-standard meanings, representations, and norms that children encounter when using math in everyday situations may not be compatible with the mathematics taught in school, and these studies suggest that differences need to be explicitly addressed in the classroom in order for everyday experiences to be used to support learning.

Perhaps one of the most interesting studies on story problem authenticity is Roth (1996), where middle-school students were engaged in an extended inquiry-based field ecology unit. As an assessment of student learning, the researchers gave students a statistics-related story problem tied to their investigations that utilized actual data they had collected. The researchers were surprised when students began to ask for more information about the story scenario, and noted that "By abstracting the problem from the environment, the range of options students had available during their field work were limited. A meaningful setting was changed into a puzzle with few options" (p. 518). This study reveals problematic nature of an assumption that any story problem could be an authentic representation of complex, situated activity.

Other research considers the ways in which using everyday reasoning when solving story problems can lead to solutions that are "wrong" with respect to the intentions of the problem developers. Ladsen-Billings (1995) describes differences in the way suburban and inner city students responded to the story problem, “It costs $1.50 to travel each way on the city bus. A transit system fast pass costs $65 a month. Which is the more economical way to get to work, the daily fare, or the fast pass?” (p. 131). Suburban students assumed that a person would commute to work 5 days a week, and concluded that the daily fare would be more economical. However inner city students opted for the transit pass, posing questions like “How many jobs does this person have?” “Do they have part-time jobs or full-time jobs?” Urban students also recognized that if the transit pass was purchased, family members could use it on evenings and weekends to go to stores, church, visiting, etc. A similar problem (with different numbers) was used by teachers who were part of the QUASAR project, and they also found students coming up with unexpected responses through the use of their everyday knowledge (Silver, Smith, & Nelson, 1995). Other research on arithmetic story problems from England has shown that working class (Cooper & Harries, 2009) and female (Boaler, 1994) students are more likely to use everyday participation practices when solving story problems, in some cases ignoring the given data and getting the problem incorrect.

Similarly, in an interview study of undergraduates solving arithmetic word problems, Inoue (2005) found that many “unrealistic” responses students gave to story problems represented unanticipated but valid interpretations of the story context based on their everyday participation practices and diverse sense-making activities. Other students in the study seemed to be conforming to the sociomathematical norms of schooling, which suggest that applying everyday knowledge to stereotypical word problems is unproductive and that focusing on calculational strategies is more appropriate. Research on elementary school mathematics has also found that when solving multiple choice problems, students focus on the answer choices rather than on making sense of the situation, and that although students may draw upon everyday knowledge when their connection with the context is strong, this knowledge can interfere with reasoning and cause students to make assumptions that are incorrect (Kazemi, 2002).

The use of traditional story problems has been framed as being consistent with modernist paradigms that “assume an unproblematic transparency of language and one-to-one matching or mapping models of the relationship between mathematical representations and ‘reality’” (Gerofsky, 2009, p. 22). Gerofsky describes how word problems are a literary and pedagogical genre, and accentuates a postmodern perspective on language as inherently ambiguous and knowledge as localized and conditional. Frankenstein (2009) discusses how word problems contain “hidden messages” (p. 111) about the sometimes taken-for-granted norms of society. For instance, a word problem about adding up prices at the grocery store contains the implicit message that it is normal to pay for food, even though many children suffer from hunger. Frankenstein calls attention to the political implications of training students to accept these scenarios as transparent and unproblematic in order to “fit in” with the larger system of norms. Other researchers argue that when domestic activities are used as contexts to provide access to mathematical ideas, these activities are subordinated to the system of school mathematics, and the specificity of the context and students’ situation-based reasoning are lost (Gellert & Jablonka, 2009).

Another line of research has investigated the impact of personalizing story contexts to students’ interests and experiences. Using a geometry unit where students explored the coordinate plane, Cordova and Lepper (1992) found that motivational story contexts targeted to students’ interests enhanced learning when compared to a control condition. Using more standard types of arithmetic word problems involving addition, subtraction, and operations on fractions, two studies (Anand & Ross, 1987; Davis-Dorsey, Ross, & Morrison, 1991) also found positive learning effects when personalizing word problems to students’ individual interests as measured by questionnaires. However, other studies have found that situational rewording intended to enrich story contexts does not lead to increased performance when compared to performance on other story problems (Cummins et al., 1988; Vicente, Orrantia, & Verschaffel, 2007). Personalized contexts may focus elementary students’ attention more closely on the situational aspects of story problems, allowing students to connect with the task, but may also lead weaker students to believe they are working the problems correctly when they are not, and can be distracting to students with lower interest in mathematics (Renninger, Ewen, & Lasher, 2002).

There is detailed knowledge in the field of how students think about arithmetic problem solving, how situational reasoning and verbal understanding affect cognition, and how the system of school mathematics interacts with problem solving. However, little similar research has been conducted for algebra.

D. Research on Algebra Story Problems

It is important to the purpose and the significance of the present work to accentuate that the research cited in the previous section was conducted with elementary or middle school students. There is a strong need to expand this line of inquiry to algebra, as story problems are a large part of curriculum, instruction, and assessment in this course, and because unique considerations relating to symbolic representations as they apply to story problems emerge at the algebra level. Further, algebra concepts are likely to be used significantly less than arithmetic concepts in day-to-day activities. Research has also pointed to the differences in language comprehension between primary and secondary students, and it has been suggested that a firmer grasp of language may deepen understanding of situational contexts while problem solving (Nathan, Kintsch, & Young, 1992; Koedinger & Nathan, 2004). Researchers studying algebra story problems have attempted to describe the processes involved in coordinating situational understanding with formal equations, the strategies used and factors of difficulty in solving algebra word problems, and the development of algebraic and symbolic competence among students.

In one of the earliest studies of algebra story problems, researchers observed that college students categorize problems based on their semantics and that they can in many cases recognize the general category a problem belongs to before reading much of the text. These students' categorizations were found to have implications for how they solved the problem; for instance, students who categorized a problem as being a "triangle" problem type while reading it used different solution strategies than those who classified the same problem as being a "distance-rate" problem type. The researchers write "the schemas directed what the subject attended to in the problem, what information he expected, what information he regarded as relevant, and even what errors he made in reading the text" (Hinsley, Hayes, & Simon, 1977, p. 103).

In another study, undergraduate computer science majors solving story problems were found to regularly use informal, non-algebraic strategies such as systematic trial and error when presented with *start unknown* algebra problems. In a start unknown problem, students are given a linear function like "y=4x+2" in a story context or as an equation, and are asked to solve for x given a specific value of y. The researchers concluded that "competent problem solving proceeds as an elaborative, interdependent exploration of two distinct problem spaces: (a) the situational context of the story problem, and (b) the quantitative constraints given explicitly or implicitly in the problem statement" (Hall, Kibler, Wenger, & Truxaw, 1989, pp. 257-258).

Other studies have shown students' tendency to use arithmetic rather than algebraic approaches to solve algebra story problems (e.g. Koedinger & Nathan, 2004), and these approaches have been attributed to a "compulsion to calculate" (Stacey & MacGregor, 1999, p. 154). Students with arithmetic-bound thinking often view variables as nonspecific referents that are not clearly defined, such that one variable could stand for two different quantities. Students also may view equations as arithmetic formulas or strings of calculations rather than statements about equality, and as a result fail to understand the utility of using an equation to solve a story problem. In a study of middle school problem solving, Humberstone and Reeve (2008) found that students' knowledge states progress from arithmetic to algebraic in several identifiable stages. They concluded that students who were able to classify algebraic equations structurally were better able to translate words into symbols, and this was demonstrative of emerging algebraic understanding.

Clement (1982) explored undergraduate students' difficulty writing an equation to go with the scenario "There are six times as many students as professors at this university," concluding that intuitive approaches often take over when students solve story problems. Successful problem solvers understand that variables stand for numbers rather than objects and are able to generate operations on these variables that create equivalence (6P=S) rather than display the literal action of the text (6S=P). Another study looked at college students' understanding of extensive (e.g. gallons) and intensive (e.g. dollars per gallon) quantities in the context of algebra word problems, and demonstrated that students make various unit-related errors when constructing equations from story problems because symbols are not well-connected to their referents (Reed, 2006). Other research has looked at algebra students' ability to write a story scenario based on a symbolic equation, finding that students view the equals sign as meaning "compute" and thus have difficulty reasoning about the structural characteristics of equations, which suggests that students manipulate symbols with little understanding of what they represent (Stephens, 2003).

Nathan and Koedinger (2000a, 2000b) and Nathan and Petrosino (2003) investigated mathematics teacher and educational researcher beliefs about story problems, and found that both groups often ranked verbally-presented problems (like story problems) as being more difficult than matched symbolic equations. However, Koedinger and Nathan (2004) found that high school students are more likely to correctly solve algebra problems written in verbal formats, including story contexts, compared to problems written as symbolic equations. This supported their *verbal facilitation hypothesis*, which stated that story scenarios provide accessibility to students because they are written in English rather than in mathematics notation. Limited support was found for the *situation facilitation hypothesis*, which stated that story scenarios provide accessibility because students are able to use everyday knowledge to assist them during problem solving. A difficulty factors assessment test was administered with problem type (traditional story problem, word equation, symbolic equation) systematically varied, and underlying mathematical structure constant.

Students in Koedinger and Nathan's (2004) study performed similarly on verbal word equations (operations written out in English) as they did on story problems.

However both of these problem types were significantly easier for students to solve than symbolic equations. A substantial number of “no response” errors on symbolic equations contributed to this result, as did students’ greater use of informal arithmetic strategies like *trial and error* and *unwind* when solving verbal problems. Story problems were significantly easier than word equations when the story used decimals in a money context, and story problems elicited greater use of the informal strategy of unwind than word equations. The unwind strategy consists of students arithmetically reversing the operations on the intercept and slope to solve a start unknown. Students’ informal strategies were found to have higher success rates than symbol manipulation strategies like equation solving.

A related study (Koedinger, Alibali, & Nathan, 2008) found that for college students, symbolic problems were easier than story problems with the same mathematical structure when the problem was an algebraic scenario with the unknown quantity referenced more than once. The researchers concluded that this more complex type of problem thwarts students’ informal solution strategies, which were considered to contribute to the verbal problem advantage in the 2004 study.

Based on their findings that story problems were not significantly easier than word equations, but that both were easier than symbolic equations, Koedinger and Nathan (2004) conclude that “contrary to views of situated cognition, this result is not simply a consequence of situated world knowledge facilitating problem solving performance, but rather a consequence of student difficulties with comprehending the formal symbolic representation of quantitative relations” (p. 129). This is among several statements in this paper, published in *Journal of the Learning Sciences,* which in part inspired the studies presented here. A situated cognition perspective would not necessarily expect participation practices from everyday situations to be brought to bear in productive ways in the stereotyped contexts of traditional story problems. Koedinger and Nathan (2004) conclude:

> One might interpret some educational innovations emphasizing story problems (e.g., Cognition and Technology Group at Vanderbilt, 1997;

Koedinger, Anderson, Hadley, & Mark, 1997), calls for mathematical reform (e.g., National Council of Teachers of Mathematics, 2000), and situated cognition and ethnomathematical research (e.g. Brown Collins, & Dugid, 1989; Cognition and Technology Group and Vanderbilt, 1990; Greeno & MMAP Group, 1998; Roth 1996) as suggesting that 'authentic' problem situations generally help students make sense of mathematics. In contrast our results are consistent with Baranes et al. (1989) that situational effects are specific and knowledge related. (p. 154)

There are several issues with this statement, the largest being the definition of authenticity. Koedinger and Nathan (2004) make an explicit comparison between the traditional story problems used in their study and other contextualized mathematics scenarios from situated cognition and ethnomathematical research (Brown et al., 1989; Cognition and Technology Group at Vanderbilt, 1990; 1997; Greeno & MMAP Group, 1998; Roth 1996). They use this comparison to question whether "authentic" contexts always help students make sense of mathematics. However, the scenarios from situated cognition and ethnomathematics research are often not similar to traditional story problems, due to their complexity, open-endedness, use of representation, and designs that take into account students' diverse sense-making activities. If the story problems from the Koedinger and Nathan (2004) study were held up to Palm's (2006) framework for authenticity, they may not fare much better than the standardized test problems shown in the introduction.

The idea that students would use participation practices from everyday situations the same way in a Jasper problem (c.f. Cognition and Technology Group at Vanderbilt, 1990; 1997), which uses rich and complex multimedia situational contexts, as in a traditional story problem does not seem likely. The Cognition and Technology Group at Vanderbilt (1990) describes anchored instruction as taking place in "macrocontexts" that allow students to explore the problem space over an extended period of time, adopt multiple perspectives, and use knowledge as a tool in ways similar to how experts in applied fields solve problems. Using a video format rather than a textual story as an anchor allows students to form rich and dynamic understandings of the situation being presented, and provides access for students with low reading skills or low knowledge of the content domain. While traditional story problems could be viewed as impoverished "anchors" for instruction, it is important that researchers gain an understanding of the affordances and constraints of different types of anchors by explicitly calling into question what constitutes contextualized mathematics.

The studies on algebra story problems cited in this section are ultimately limited for the research purposes here, as they are not framed to critically examine the multiple, interacting, complex social systems that act upon students from a situated cognition perspective. These systems include the system of school mathematics, the out-of-school systems that each student participates in, and even the system that authored the story problems that the student is working on. Roth (1996) observes "Word problems as text share with instructions that, in order to understand them correctly, readers have to share the author's common sense; this would allow them to understand all that which goes without saying" (p. 519). There is a strong need for research on story problems at the secondary level that takes into account situated views of problem solving and the impact of sociomathematical norms.

E. Models of Problem Solving

Nathan et al. (1992) proposed a model of algebra story problem comprehension, based on the idea that when solving word problems students must coordinate three levels of representation: (1) the *textbase* or the textual information in the problem (2) the *situation model* or mental representations of the relationships, actions, and events in the problem, and (3) the *problem model* or mental representations of formal algebraic structure involving variables, equations, etc. The formation of the situation model draws upon students' everyday knowledge to "fill in the gaps left by a sparse story," (p. 333) and is a relative, qualitative representation of the story's action. The situation and problem models are thought to be mutually supportive, with students iteratively moving between representations. Thus situational understanding of the problem scenario is tied to and can scaffold symbolic representations that are accurate and meaningful. This is contrasted with *translation-based* approaches, where students bypass the formation of a situation model and attempt to directly translate a problem text into formal expressions that are inconsistent with the situation being described. Figure 4 depicts this model for story problem comprehension within the specific context of an interactive algebra tutoring program ANIMATE, which was designed to scaffold the construction of situation models through animated simulations. This framework offers leverage in interpreting students' problem-solving practices as they negotiate text, described action, and mathematical formalisms. This model will be drawn upon in the analysis of students' problem-solving behaviors presented here.

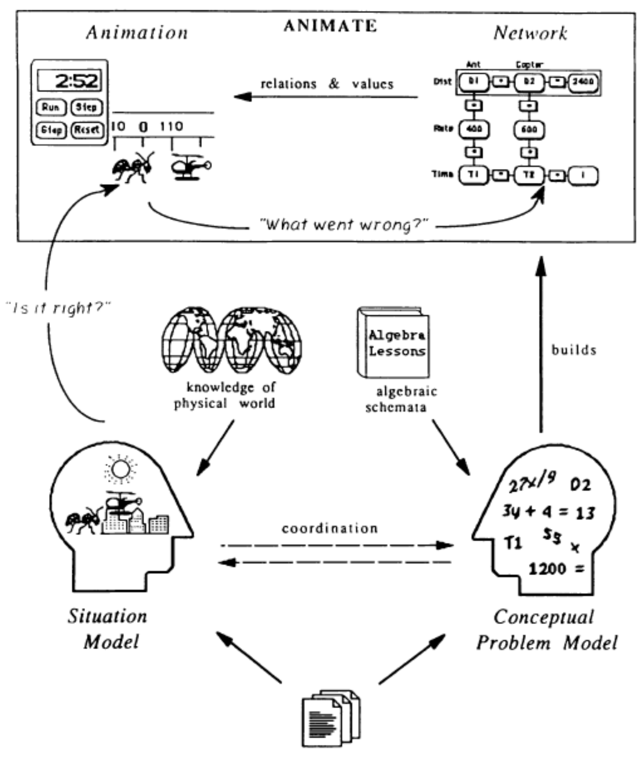


Figure 4. Graphic from Nathan, Kintsch, & Young (1992) p. 347, showing proposed model of algebra word problem comprehension, in the context of their study on the ANIMATE learning environment. Reproduced with permission of author.

The observation that students sometimes use direct translation approaches to solve story problems by using keywords and superficial solution strategies is also present in the literature on arithmetic story problems (Hegarty et al., 1995; Palm, 2008). Direct translation approaches will be referred to here as *non-coordinative* approaches, meaning students construct a problem model from the textbase without coordinating this problem model with situational understanding of the given problem, or an elaborated situation model.

It is interesting to compare the Nathan et al. (1992) model in Figure 4 to a more general model of word problem comprehension based primarily on arithmetic research, proposed by Greer (1997). As shown in Figure 5, in addition to students' "knowledge of the real world" contributing to formation of the situation model, Greer includes "implicit grasp of rule of 'word problem game'" as also mediating situational understanding. This addition allows for the explicit consideration of the complex, social system of school mathematics as mediating students' actions, an element that is missing from past studies of algebra story problems.

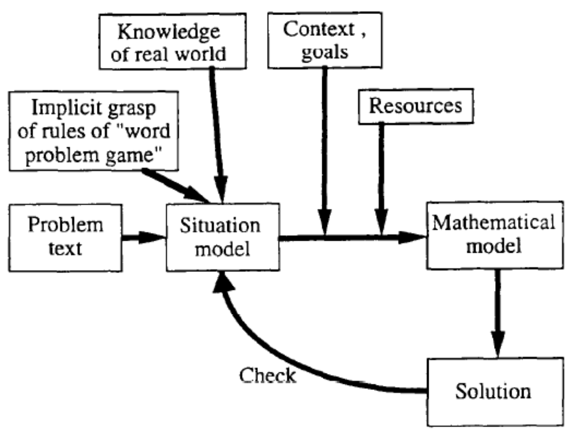


Figure 5. Graphic from Greer (1997) p. 301 showing proposed "minimal schematization" of word problem solving. Reproduced with permission of author.

F. Research on Symbolic Representations

Situated approaches view mathematical representations, like symbolism, as interpretive conventions embedded in social activity and intended to be used as tools to promote participation (Brown, Collins, & Dugid, 1989; Greeno, 1997). As Greeno and Hall (1997) write:

> Learning to construct and interpret representations involves learning to participate in the complex practices of communication and reasoning in which the representations are used. This learning involves much more than simply learning to read and write symbols in arrangements corresponding to the accepted forms. (p. 361)

However, as these authors and others (Lesh & Harel, 2003; Reusser & Stebler, 1997) note, the use of symbolism in school mathematics tasks often does not resemble authentic uses of representation; indeed, in many tasks, including story problems, symbolism is viewed as an end in and of itself. Students may not be given the opportunity to compare the trade-offs of different representational forms, or understand representations as being constructed and adapted according to the local purposes of the problem-solver (Greeno & Hall, 1997). Greeno (1991) describes how representations of concepts, like symbolism, are often confused with the concepts themselves, and that representations “should not replace experience in conceptual environments” (p. 177). Greeno (1991) discusses how reliance on representations rather than experience may cause students to interact with symbolic representations without understanding the conceptual entities they signify.

Similarly, Schoenfeld (1988) describes how processes of formal mathematics like symbolism are disconnected from the situated, real world objects they represent in school mathematics tasks, and are presented in such a way that they have little relation to mathematical discovery or invention. Lesh and Zawojewski (2007) discuss how, in applied problem solving, people are not likely to develop a general conceptual tool like a symbolic representation for a problem that is only going to be solved a single time, like a traditional story problem.

Mark and Koedinger (1999) studied progressive abstraction approaches in algebra by presenting students with traditional story problems on linear functions where they solved *result unknowns*. In result unknown, students are given a linear function like “y=4x+2” in a story context or as an equation, and they are asked to solve for y given a specific value of x. The students in Mark and Koedinger’s (1999) study then used these “concrete cases” to generalize a symbolic expression. The study found that although students preferred to conceptualize result unknowns in arithmetic rather than symbolic form, students were usually able to symbolize usually after just one or two concrete cases.

Gluck (1999) examined where students’ eyes were focusing as they solved such progressive abstraction result unknown problems, and found that at least 46% of the time, after constructing a symbolic expression students did not look at the expression to solve a related *start unknown* (solve for x given this y). This suggests that students may be scaffolded into constructing a symbolic equation from concrete cases without adopting the uses of that representation that are valued in a school mathematics context (i.e. for equation solving). Arcavi (1994) in his work on algebraic *symbol sense* finds:

> Many high school students make little sense of literal symbols, even after years of algebra instruction. Even those students who manage to handle the algebraic techniques successfully often fail to see algebra as a tool for understanding, expressing, and communicating generalizations, for revealing structure, and for establishing connections and formulating mathematical arguments (proofs). (p. 24)

Arcavi (1994) accentuates that symbolic competence includes knowing when symbolic representations are useful problem-solving tools, and when they should be abandoned in favor of other methods for representing the problem. Symbolic competence also includes an understanding of the meaning of symbolic representations and understanding why automated symbolic manipulations “work.” Finally, Arcavi argues that algebraic symbols should be presented to students as powerful tools to solve, understand, and communicate about problems, rather than as “formal and meaningless entities” (p. 33).

Also important to a discussion about symbolism is how a variable is conceptualized when learning algebra. Traditionally, a variable is viewed as a specific, unknown value or a “missing number,” and solving an equation in one variable is conceptualized as finding the value of the variable for which the number sentence is true. However, in a “functions-based” approach, a variable is viewed as a set that can take on many possible values, and solving an equation in one variable is finding the shared domain for which the functions on either side of the equals sign have the same output (Chazan, 1999).

Research has also identified several different ways a function can be conceptualized in algebra. An *action* conception of a function is characterized by viewing a function as a “repeatable mental or physical manipulation of objects” (Bridenbach, Dubinsky, Hawks, & Nichols, 1992, p. 251) for which there is an explicit formula. From this perspective, a function is a string of operations performed one step at a time, and calculating the value of a function for one x value is disconnected from finding the value of the same function for a different x value. A *process* conception of a function is characterized by viewing a function as a systematic process that transforms elements in the domain to elements in the range as a complete and dynamic activity. Students working from a process perspective may understand operations such as composing or inverting functions, since they view a function as a systematic relationship between inputs and outputs (Bridenbach et al., 1992). An *object* conception of a function is characterized by viewing a function as a part of class of functions based on its parameters, and manipulating and understanding algebraic expressions as objects in their own right (Moschovich, Schoenfeld, & Arcavi, 1993; Sfard, 1991).

Sfard and Linchevski (1994) make the case that verbally-presented problems perpetuate action and process (operational) ways of thinking, which makes the transition to object (structural) ways of thinking difficult. They also point out that writing a general equation to solve a word problem may be consistent with the development of a structural conception of a function rather than a purely operational conception, as calculations are being suspended for the sake of representing a general algebraic object.

However, developing multiple perspectives on the meaning of a function is difficult for mathematics teachers as well as students. Sherman and Greeno (2010) report on a teaching experiment where pre-service teachers were immersed in discussions about different conceptions of functions and variables, as well as discussions related to formation of problem and situation models. They found that teachers struggled to synthesize these different theoretical ideas into a general, coherent model of student understanding.

One important approach to developing symbolic representations that are tied to purpose, meaning, and students' cultural practices can be found in the curriculum used by the Algebra Project (Moses & Cobb, 2001). The learning trajectory used in the curriculum begins with students experiencing a physical event, like a trip on the metro, which is modeled using progressively more abstract representations. Students first draw pictorial representations of features of the event that are of value to them, while discussing the event in their own language ("people talk"). Students move towards more structured language ("feature talk") which encodes or isolates the mathematical features of the event, and then based on their pictures, models, discussions, and writings, they begin to construct symbolic representations. Symbolic representations are initially private constructions of their individual student creators, but the class proceeds to negotiate a more general symbolic system through shared discourse (Moses & Cobb, 2001).

Lehrer and Schauble (2006) discuss the importance of modeling for engaging students in "the invention and revision of systems of inscription – ways of representing – the natural world" (p. 176). In the domain of mathematics, they conducted a study where elementary students progressively mathematized systems of classification for drawings through several modeling cycles (Lehrer & Schauble, 2000). This is similar to the *models and modeling perspective* where students collaboratively model and mathematize complex scenarios that encourage the creation of powerful representational systems with conceptual tools that are reusable, modifiable, and shareable (Lesh, Cramer, Doerr, Post, & Zawojewski, 2003). These types of progressive abstraction approaches, tied inexorably to meaning, experience, culture, and purpose, stand in contrast to cognitive notions of gaining symbolic competency through repeated practice.

G. Purpose of Learning Algebra

Cobb et al. (2001) distinguish between three types of mathematical norms: (1) norms surrounding the ways in which tools and symbols are used and reasoned with, (2) norms surrounding mathematical argumentation, and (3) norms surrounding the purpose of the mathematical activity. The purpose of learning algebra is not always transparent to teachers, learners, or even curriculum developers. Chazan (1999) wrote about his experiences teaching algebra using a traditional curriculum, describing how he felt the course consisted of a long list of techniques and was focused inwardly with each topic being justified only with respect to future coursework. He descrbies how "I myself could not see connections between the algorithms I was teaching and the activity of the people in the world around me. These algorithms were primarily useful in solving problems in school or on academic tests" (p. 125).

In an Algebra I textbook evaluation by the American Association for the Advancement of the Sciences (2000), less than half of the 12 textbooks reviewed received a satisfactory or higher rating on "Conveying Unit Purpose," and only 1 of the 12 was satisfactory for "Conveying Lesson Purpose." However, every single textbook scored satisfactory or higher on "Providing a Variety of Contexts" and every textbook except one scored satisfactory or higher on "Providing Firsthand Experiences" (AAAS, 2000). This evaluation may be interpreted as suggesting that although textbooks are filled with many contextualized story problems, these problems are disconnected from the actual purpose and utility of learning algebra.

H. Summary

This chapter concludes with a summary of the salient points of the literature review that have direct bearing on the current set of studies. Situated research in mathematics education has shown that set of sociomathematical norms and mathematical practices mediates students' participation in school mathematics (Cobb & Bowers, 1999). Research on students' participation in mathematics classes reveals that many students hold problematic beliefs about the nature of mathematical activity (Schoenfeld, 1988).

For instance, students may use cue-based and direct translation strategies to solve problems, which are tied to the immediate context of school and disconnected from problem solving in other systems of participation (Boaler, 1998; Hegarty et al., 1995). As the system of schooling has its own set of participation structures, the way in which practitioners use mathematics differs from how mathematics is used in school (Lave & Wenger, 1991; Masingila et al., 1996; Resnick, 1987; Saxe, 1988; Taylor, 2005). Specifically, representational tools like symbolism are often framed in school mathematics classes as ends in and of themselves, and are used in unrealistic ways (Greeno & Hall, 1997; Schoenfeld, 1988).

Story problems are perhaps the most ubiquitous way in which mathematics is contextualized or made "applied" in many classrooms today. However, research has shown that students struggle to solve non-standard arithmetic story problems (Carpenter et al., 1983), and often do not realize when story problems are impossible to solve from the given information (Palm, 2008; Reusser & Stebler, 1997; Xin, 2009). Further, slight variations in wording of story problems, as well as the interaction of the story context with problem structure, can affect strategy use in arithmetic word problems (Baranes et al., 1989; Carpenter & Moser, 1984). In algebra, the way in which students conceptually categorize problems while reading them also affects the solution strategies they use (Hinsley et al., 1977).

Nathan et al. (1992) proposed that when solving story problems, students form a propositional textbase and then coordinate situation and problem models. However, formation of mental models of situational contexts may be problematic for young students who struggle with text comprehension (Baranes et al., 1989). Further, using everyday knowledge to scaffold the formation of a situation model can be disruptive to problem solving when this knowledge is inconsistent with the norms and expectations of the problem situation (Inoue, 2005; Kazemi, 2002; Ladsen-Billings, 1995). Thus students' understanding of the rules of the "story problem game" may mediate problem-solving success in the social system of school mathematics (Greer, 1997).

A number of studies have shown that students regularly use informal, arithmetic strategies to solve algebra story problems, like trial and error approaches (Hall et al., 1989; Koedinger & Nathan, 2004; Stacey & MacGregor, 1999). Students also have difficulty coordinating symbolic representations with story scenarios (Clement, 1982; Reed, 2006; Stephens, 2003). Research has demonstrated that verbally-presented problems like story problems or word equations are easier for beginning algebra students to solve than matched symbolic equations (Koedinger & Nathan, 2004). This result has been shown to be contrary to the beliefs of many teachers (Nathan & Koedinger, 2000a, 2000b; Nathan & Petrosino, 2003) and the organization of many textbooks (Nathan et al., 2002).

Other research has explored the impact of personalizing story problems to students' interests and experiences or enriching the situational contexts of story problems. However, the results are mixed with some studies showing positive effects (Anand & Ross, 1987; Davis-Dorsey et al., 1991) and others showing no effect (Cummins et al., 1988; Vicente et al., 2007). Further, research on personalization has been conducted primarily with elementary school students.

In the next two chapters, the pilot studies that led to the main dissertation study on algebra story problems are discussed. The first pilot study describes how research interest in story problems arose from video observations of an algebra class learning to use a computer-based curriculum that accentuated modeling "realistic" story scenarios with different mathematical representations. The second pilot study presents a rich, qualitative analysis of 5 algebra students solving story problems in an interview setting, focusing on the sociomathematical norms surrounding problem solving.

III. Chapter Three: Pilot I

The first pilot study (DiBiano & Sabouri, 2006) was conducted in fall of 2006 in a regular-level Algebra I classroom at the urban Texas high school that is the site of research throughout this series of studies. Cognitive Tutor Algebra, a computer-based intelligent tutoring system for Algebra I, was currently part of the adopted curriculum in the school district, and the broad goal of the first pilot was to study its implementation at a low-performing school. Two class periods of students using Cognitive Tutor Algebra were videotaped, and interviews were conducted with algebra teachers in the district as well as the district representative for Carnegie Learning, the company that produces Cognitive Tutor Algebra.

A. Background of Cognitive Tutor Algebra

Cognitive Tutor Algebra is an intelligent tutoring system developed in the 1990s based on John Anderson's theory of acquisition of cognitive skills, the ACT Theory of Learning. In the Adaptive Character of Thought (ACT) theory, all knowledge can be divided into declarative knowledge and procedural knowledge. Declarative knowledge is represented by describable facts, while procedural knowledge is manifested through performance. The ability to speak English would be procedural knowledge, and the unit of procedural knowledge is the *production rule*. The ACT theory states that all tasks involve a combination of declarative and procedural knowledge, with declarative knowledge becoming proceduralized through repeated practice (Anderson, 1993).

The Cognitive Tutor for Algebra software is an interactive computer environment that presents students with multi-part algebra story problems. The program incorporates the ACT theory of learning by using both a *model- tracing approach* and a *knowledge-tracing approach*. In the model-tracing approach, the computer software relates the students' problem-solving actions back to a cognitive model, and uses this comparison for individualized error feedback. In the knowledge-tracing approach, the computer software tracks the student's learning from one problem to the next in order to identify the students' strengths and weakness in terms of production rules. The software then uses this analysis to individualize the pace of the instruction and the selection of problems (Koedinger, Anderson, Hadley, & Mark, 1997). Problems presented in the Cognitive Tutor software are designed to be "real-world" and "culturally or personally relevant to students" (Koedinger, 2001, p. 155). Koedinger et al. (1997) describe Cognitive Tutor problems as follows:

> In the PUMP classroom, students work on mini-projects investigating problem situations like comparing the current quality and growth rate of old growth forests in the U.S. to the harvest rate. Students investigate such situations by (1) addressing questions like, "Assuming these figures do not change, when will all the old growth forest be gone?", (2) creating a table to investigate the relationships between quantities (3) scaling, graphing, and identifying points of intersection, (4) using algebraic notation to concisely represent the underlying structure of the situation, and (5) using algebraic notation to compute solutions. (p. 32)

A screen shot of a typical Cognitive Tutor problem is included in Figure 6.

Cognitive Tutor problems fit relatively well into the norms and practices of the "school mathematics" system, which may be in part the reason for the widespread adoption of this curriculum. However, there are some important differences between Cognitive Tutor problems and traditional story problems. Cognitive Tutor problems tend to take longer to solve, since they have several related parts, and they make explicit connections between the representational forms of tables, graphs, and symbolic equations.

Studies support the claim that use of the Cognitive Tutor Algebra curriculum improves student learning relative to control groups (Koedinger et al., 1997; Koedinger & Sueker, 1996; Morgan & Ritter, 2002). In addition, research suggests positive effects of Cognitive Tutor on motivation (Anderson, Corbett, Koedinger, & Pelletier, 1995), attitudes towards mathematics (Morgan & Ritter, 2002), and student-student collaboration (Anderson et al., 1995; Bransford, Brophy, & Williams, 2000).

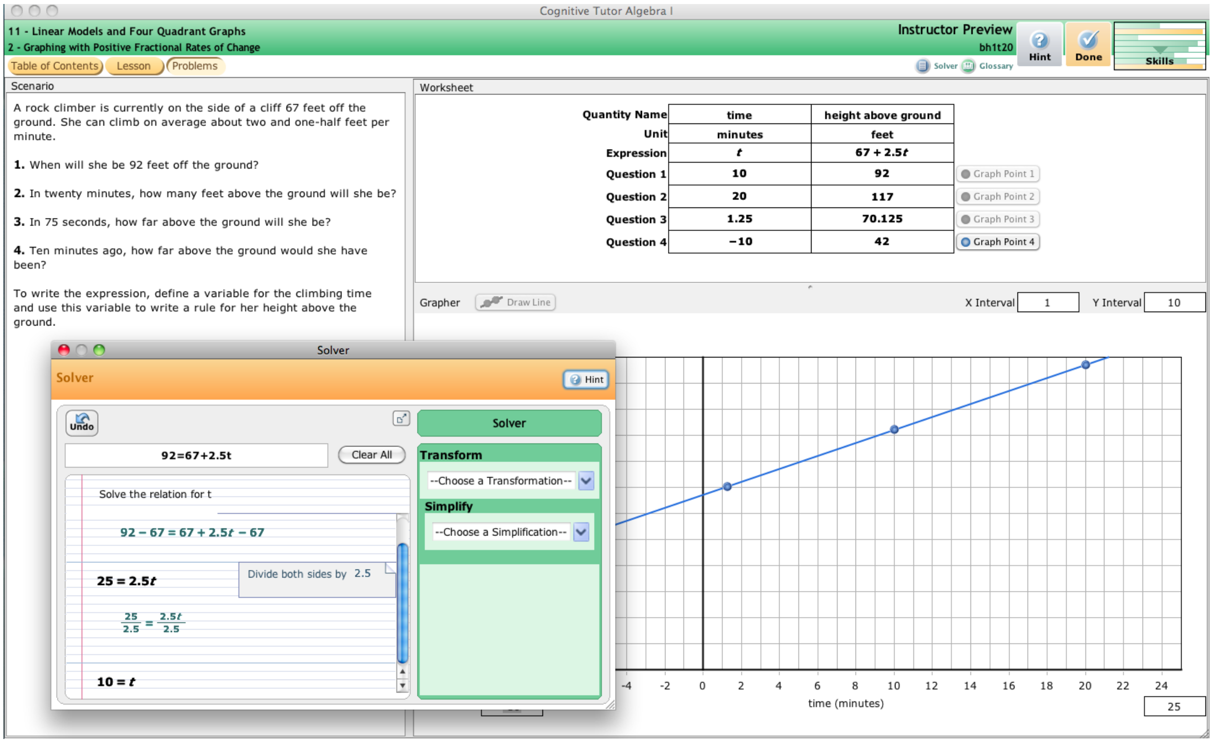


Figure 6. Screenshot from Cognitive Tutor Algebra curriculum, distributed by Carnegie Learning (http://www.carnegielearning.com/). Used with permission of Carnegie Learning.

B. *Research Questions and Methods*

The purpose of the first pilot study was to build on current research detailing teacher-student interactions in a Cognitive Tutor classroom in order to look at the efficacy of claims in the literature relating to student motivation, student collaboration, problem relevancy, and pedagogical approaches. More specifically, the research questions were:

P1.1 What types of interactions does the teacher engage in with the students as they work through the Cognitive Tutor software?

P1.2 What are the issues that urban schools struggle with when implementing a software-based educational innovation?

At the high school site where this study took place in 2006, the majority of the students (56%) were Hispanic, with 34% White non-Hispanic, 9% African American, and 1% Asian/Pacific Islander. Economically disadvantaged students made up 48% of the school's enrollment, and 8% of students had limited English proficiency. The school had been rated "Academically Unacceptable" by the Texas Education Agency for its performance in the 2005-2006 school year. Two classes of a first-year algebra teacher were videotaped. The teacher had been selected to participate in this study by the math department head because she was a strong supporter of the Cognitive Tutor software, and was currently the only teacher at the school implementing the program. The teacher will be referred to as "Mrs. A," and there were 12 students in the observed class.

The video footage was transcribed and analyzed using an expanded version of the coding scheme Heffernan (2001) designed for his study of human algebra tutor interactions. Around 600 teacher and student utterances from the two video observations were coded (not counting utterances of off-topic conversation).

C. Results

Results from the discourse analysis showed that the most common utterance by the teacher was to ask the student to supply information not directly in the problem by questioning or hinting, and the most common utterance from a student was to give the correct answer to the teacher. While the teacher questioned the students frequently, most of her questions were repetitive, and there was a noticeable lack of more sophisticated questioning techniques and giving students feedback. The coding also revealed that the teacher often set sub-goals for the students instead of encouraging them to figure out the sub-goals for themselves. And while the teacher made some minor efforts to assess what students were thinking, the teacher's major focus was getting the students to type in the correct answer, rather than helping students to conceptually understand the problem.

There were also several instances where the teacher told students to slow down and stop putting in answers before thinking about the problem, which may correspond to other Cognitive Tutor research on "gaming the system" (Baker, Corbett, & Koedinger, 2004). As a whole, the analysis of the video footage supported the idea that the norms of a traditional mathematics classroom were still in place in this classroom, even with the use of a learning science innovation like Cognitive Tutor Algebra.

The teacher made various comments as students worked through the software. In terms of the wording of the problem scenarios, Mrs. A spontaneously mentioned the following after a student struggled to understand a story about an "automobile plant":

> It's funny because some of them are like… it says automobile plant, and she was thinking plant plant. And some of it's just like, a lot of it's the word, it's stuff they haven't heard before. Like we had a math problem that asked about a greenhouse, the area of a greenhouse, and they've never heard of a greenhouse, they don't know what a greenhouse is. So then people get stuck on that in the problem, what's a greenhouse, and not the actual math. So sometimes it's vocabulary, not math. That's a big problem with the TAKS test, because there's a lot of vocabulary they haven't heard of on there. (Mrs. A, November 22, 2006)

The teacher also favored the approach of the software which required students to construct a symbolic representation after several concrete cases:

> So it's nice because it forces them to come up with independent and dependent quantity and the expression on their own. That's what they need to do ….because they have to keep doing it and it tells them if it's right. (Mrs. A, November 22, 2006)

The above quote also alludes to the requirement of the Cognitive Tutor software that students name or label the independent and dependent quantities in each scenario (i.e. hours worked, money earned). The labeling of the quantities caused continuous issues for students during the observations (12 distinct exchanges); students seemed to have trouble using the stories to figure out the two relevant quantities involved in the rate of change, and then figure out which depended on which. At one point when a student complained that the problems were "hard," the teacher asked whether the math was hard, or whether thinking of the words for the independent and dependent quantities was hard – the student responded that the words were the hard part, with another student adding that the "math" part was easy.

There were also issues with the way the students worded and entered in the labels for the independent and dependent quantities, with the teacher complaining "This is the one thing that I don’t like about this, is because it’s so particular [to the] problem and you write essentially the exact thing, and it says ‘No I want this.’ You know what I mean? It’s very specific.” (Mrs. A, November 22, 2006). This conception of the software program as “wanting” something was present throughout the teacher’s interactions with the students (14 instances). Teacher utterances like “It wants you to find those answers from the graph” and “It won’t let you type it until you put it on the graph” (Mrs. A, December 1, 2006) show that while working on Cognitive Tutor, rather than students and teachers determining sociomathematical norms relating to problem sub-goals, encouraged strategies, and solution formats, the computer program was in part responsible for determining these norms.

D. Discussion

Many of the issues discussed in the preceding section, as well as the review of the literature, suggest that the use of the Cognitive Tutor Algebra program requires students and teachers to learn to participate in a specific social system for the learning of mathematics, whose norms include, among others:

1) A level of vocabulary that the creators view as appropriate for Algebra I students
2) The use of problem situations that the creators view as being accessible and relevant to Algebra I students, and the idea that there is some general benefit to embedding algebra concepts in story scenarios
3) The idea that concrete cases scaffold students to abstraction, and that students should be required to construct symbolic representations for story problems
4) The use of specific strategies to solve algebra problems that are encouraged by the program’s hints, feedback, and tools
5) The requirement that students should explicitly label the independent and dependent quantities in each problem, and that these labels should conform to a specific format

The first four norms on the above list were identified in this initial pilot study, and became the basis for both the second pilot study and the dissertation study reported here.

In addition, the observations in the first pilot study suggested that students struggle with story problem comprehension, and may not develop a fully elaborated understanding of the given situation before trying to solve the problem. Thus some students resorted to strategies that bypassed situational sense-making of the story. One of these strategies seems to be getting direct assistance from the teacher, with another perhaps being some form of "gaming the system," or entering in answers quickly and repeatedly to see if the software accepted them. The idea that students may not be reasoning deeply about the given story context when confronted with traditional story problems also became a focus of both the second pilot study and the dissertation study.

With respect to the second research question, ("What are the issues that urban schools struggle with when implementing a software-based educational innovation?") the pilot study found that there were multiple issues working against teacher implementation of this software at the school site, including lack of access to computer labs and computers, administration failing to get the program installed and running, student behavior and management issues, and continuous computer breakdowns. An interview with Carnegie Learning's sales representative (Donna Black, personal communication, 2006) revealed another issue with implementation – she described how teachers struggle to modify their pedagogical approaches to make full use of the software's resources. The representative discussed how teachers have to go from a "sage on stage" to a "guide on side," and one of the conclusions from this pilot study was that the short, 2-day professional development given to Mrs. A, aimed mainly at having teachers understand the software, may not be sufficient.

These findings were consistent with Cuban, Kirkpatrick, & Peck's (2001) conclusion that high school teachers use technology in their classrooms infrequently and in a relatively limited manner, and that the norms of teacher-centered instruction are rarely disrupted by technology. Cuban et al. attribute this result to issues with technology breakdowns and maintenance issues, as well as the norms and organization of secondary schooling. Many schools have short, 50-minute periods for instruction, and provide insufficient time for teachers to collaborate around learning to use technology innovations. Initiating change in school systems is difficult, as many stakeholders are accustomed to the “status quo.”

Collins and Halverson (2009) frame the resistance of school systems to technological integration as a result of the incompatibility between school and technology. For instance, in the school system, the norm is *uniform learning*, or that everyone should learn the same thing, while technology promotes *customization* that responds to the needs and interests of different learners. Further, the school system places the teacher as the expert; with technology, diverse sources of expertise are valued. Collins and Halverson also describe how school promotes “just-in-case” learning, while technology promotes “just-in-time” learning. They make the argument that if schools cannot adapt to technological advances, education may begin to primarily take place outside of school.

Although the first pilot study’s findings with respect to use of technology in schools were interesting, the second pilot study was primarily focused on further investigating the norms surrounding story problems identified here. The first pilot study suggested that students struggle to interpret word problems and may use non-conceptual strategies that bypass situational understanding of the given story. Thus the idea that contextualizing mathematics is beneficial for students because it provides access or allows transfer is not as simplistic as the conventional wisdom of schooling may suggest. In the first pilot study, students in the Cognitive Tutor classroom were adopting a powerful set of sociomathematical norms that mediated their problem solving. In the second pilot study, the norms surrounding story problems were further elaborated by giving five algebra students several Cognitive Tutor story problems to solve using pencil and paper.

IV. Chapter Four: Pilot II

After the first pilot study was completed, I attended the 2007 Pittsburgh Science of Learning Center (PSLC) (www.learnlab.org) Summer School workshop on technology-enhanced learning experiments and Intelligent Tutoring Systems. As a result of this visit and conversations with researchers at the workshop, I wrote a proposal to the PSLC to conduct several additional studies on algebra problem solving. The PSLC was specifically interested in investigating an altered version of Clark and Mayer's (2003) *personalization principle*. The original principle stated that tailoring instruction to students' typical pattern of language use would enhance learning. However, the PSLC was also interested in researching whether personalizing problems to individual students' interests and experiences would enhance learning, especially at the secondary level. This fit well into my broader research agenda to look at the affordances and constraints of story problems as contextualized mathematics, and to explore the sociomathematical norms surrounding the solving of story problems while focusing on symbolic representations. My research proposal was accepted by the PSLC with funding in summer of 2008. The second pilot study was conducted in fall of 2008 at the same urban Texas high school that was used in Pilot I and that is used for the dissertation study.

A. *Research Questions and Methods*

In the second pilot study, the main research questions being investigated were:

P2.1 How do students use participation practices from everyday situations to scaffold problem solving when given algebra story problems?

P2.2 How does personalization of problem scenarios to student interests and experiences affect problem solving?

A teacher at the high school site who the primary researcher was familiar with was recruited to participate in the second pilot study as well as the dissertation study; she will be referred to as "Mrs. C." Mrs. C was a fourth-year teacher from a university certification program who had been teaching Algebra I courses at the school site since she began teaching. During the year of the second pilot study and dissertation study, Mrs. C had three regular-level Algebra I classes with approximately 74 total students enrolled. Five students from Mrs. C's classes volunteered to participate in the second pilot study for a small stipend.

Each student was engaged in a 15-30 minute entrance interview where they were asked questions about their lives and interests, similar to the questions provided in the interview protocol in Appendix A. The purpose of this interview was to determine what out-of-school topics students were interested in, in order to write personalized algebra story problems that corresponded to these interests. Based on the entrance interview, four problems were written for each student. Two of these problems were Cognitive Tutor Algebra problems with stories that were modified to be personalized to students' out-of-school interests, one was a standard story problem from the Cognitive Tutor Algebra curriculum, and one was a generic version of a Cognitive Tutor problem with general referents and simplified language. There were five base problem scenarios from Cognitive Tutor Algebra that were used in this study, which are shown in Table 1. Modified versions of each of these problem scenarios were written to be either personalized or generic; an example of how these modifications were done is shown in Table 2. The order in which students were presented problems of various types (personalized vs. normal vs. generic) was randomized.

Once the four problems had been written, each student was engaged in problem-solving interview lasting 30 to 45 minutes. The interview methodology was a semi-structured interview, which Kvale (1996) describes as "an interview whose purpose is to obtain descriptions of the life world of the interviewee with respect to interpreting the meaning of the described phenomenon" (pp. 5-6). Each student's problems were written in advance and matched in terms of mathematical structure with the problems other students would be solving. Students were asked to think out loud as they solved their problems, and were explicitly instructed to construct self-explanations (Chi & VanLehn, 1991). Beyond probes reminding students to explain their thinking, the interviewer made only occasional use of spontaneous questions. The interviewer sometimes asked clarifying questions during the interview, and asked two pre-determined questions when the interview was complete: 1) Which problem was easiest? What do you think made it easy? and 2) Which problem was hardest? What do you think made it hard?

Problem Text
1) A machine called the Crawler, which moves space shuttles, travels at the rate of 3 feet per second. The Crawler is currently 10 feet from the hanger, moving toward the launching pad. a. How far will the Crawler be from the hanger in 30 more seconds? b. How far will the Crawler be from the hanger in two more minutes? c. Write an algebraic expression for the total distance from the hanger as a function of time. d. In how many more seconds will the Crawler reach the launching pad, which is a total of 600 feet from the hanger?
2) An international team of explorers plans to attempt the longest surface crossing of the Arctic Ocean in a single season. They hope to leave Russia in March and reach Canada by July - nearly three months later. To complete the 1,800 mile trip, they must average twenty miles per day, traveling in dogsleds and special canoes designed for ice-choked waters. a. How far will they travel in five days? b. After thirteen days, how far will they have traveled? c. Write an algebraic expression for distance they've traveled as a function of time. d. How far will they travel in twenty days?
3) An experimental liquid (LOT#XLHS-240) is being tested to determine its behavior under different extreme temperatures. Its current temperature is 11 degrees Celsius and is slowly being increased by 2 degrees per minute. a. What will the temperature of the liquid be ten minutes from now? b. What will the temperature of the liquid in half an hour? c. Write an algebraic expression for the temperature of the liquid as a function of time. d. When will the temperature be one hundred degrees Celsius?
4) A company has been created to produce a new product. The company predicts that its capital expenditure (the one-time start up costs to buy supplies, equipment etc.) will be $500. It plans to sell its new product for a profit of $10 per unit. (The profit per unit is the price at which it sells each unit minus the costs to make and sell each unit.) The company's profits for its first year of operation will be its total profits from sales minus its capital expenditure. a. If the company sells 190 units during the first year, how much total profit will the company make? b. How much total profit will the company make if it sells one hundred and fifty units? c. Write an algebraic expression for profit as a function of the number of units sold. d. How many units will the company have to sell to break even?
5) A skier noticed that he can complete a run in about 30 minutes (half an hour). A run consists of riding the ski lift up the hill and skiing back down. a. If he skis for three hours, how many runs will he have completed? b. If he skis for six hours, how many runs will he have completed? c. Write an algebraic expression for the total number of runs as a function of time. d. If he plans on making 11 runs, how many hours will he have to ski?

Table 1. Base problems used for Pilot I study, taken from the Cognitive Tutor Algebra curriculum

Type of Problem	Example
Normal Story Problem	A skier noticed that he can complete a run in about 30 minutes (half an hour). A run consists of riding the ski lift up the hill and skiing back down. a. If he skis for three hours, how many runs will he have completed? b. If he skis for six hours, how many runs will he have completed? c. Write an algebraic expression for the total number of runs as a function of time. d. If he plans on making 11 runs, how many hours will he have to ski?
Personalized Story Problem (Student discussed watching Southpark during entrance interview)	You're watching a Southpark marathon on Comedy Central. Each episode of South Park is 30 minutes long. a. If you're watching the marathon for 3 hours, how many episodes have you watched? b. If you're watching the marathon for 6 hours, how many episodes have you watched? c. Write an algebraic expression for total episodes watched as a function of time. d. If you end up watching 11 episodes of Southpark, how many hours did it take?
Generic Story Problem	A person completes a task in 30 minutes. a. How many times can they complete the task in 3 hours? b. How many times can they complete the task in 6 hours? c. Write an algebraic expression for the total number of times they can complete the task as a function of time. d. If the person completed the task 11 times, how long did it take?

Table 2. Example modifications on base problem in Pilot II study

The interviews were audio recorded and transcribed. As this was a pilot study, it was intended to be primarily exploratory, with the purpose of figuring out the right questions to ask, rather than to determine answers or form conclusions; the focus was therefore on discovery of new dimensions of the topic under study (Kvale, 1996). It was also important to develop a more detailed procedure for conducting the interviews that could be used in the larger follow-up study. The remainder of this section uses narratives to discuss the five pilot interviews in some detail; all student names used in the discussion are pseudonyms.

A. *Interviews with Amy*

Amy was a 9th grade Hispanic student taking Algebra I with Mrs. C. During her entrance interview, she discussed how she enjoyed being part of the school band and playing the snare drum, and described how she takes trips and participates in activities related to being in the band. Two personalized problems were written for Amy based on the entrance interview; in one problem Amy was with the school band marching across the football field at a constant rate, and in the other she was helping to organize a fundraiser for the band.

During the problem-solving interview, Amy provided a response for each part of all four questions she was given, but had issues with some problem parts. Her first context was a personalized version of problem 1 from Table 1, reading, "You are marching at Texas High band practice at a rate of 3 feet per second. You are 10 feet away from the school, marching towards the football field." The first result unknown problem Amy answered in this context was "How far will you be from the school in 30 more seconds?" Amy initially seemed unsure of the answer, with the following exchange occurring:

> Amy: 30 seconds… (long pause - 19 seconds). I guess it would be 90 feet away?
>
> Interviewer: OK. What part was confusing you, do you think?
>
> Amy: Because I think I was thinking like we were marching toward the school. And in position feet -oh wait, and then you add the 10, and so it's 100 feet away from the school.

In retrospect, both the version of this problem in the source curriculum and the personalized version were ambiguous. In order to obtain the intended answer to this problem, the assumption must be made that the marcher's path is perpendicular to the school (or the hangar in the original problem). For instance, the situation model shown in Figure 7, although technically matching both statements of the problem, would not lead to the intended solution. Whether the ambiguous problem statement mattered for Amy and caused some of her confusion is not clear. The bigger issue is that story problems like this ask students to accept an implicit assumption to interpret the scenario in the most simplistic, unproblematic way possible, in order to calculate a precise numeric answer. Solving such story problems requires that students accept the stereotyped nature of school mathematics tasks, and put aside considerations related to their everyday experiences.

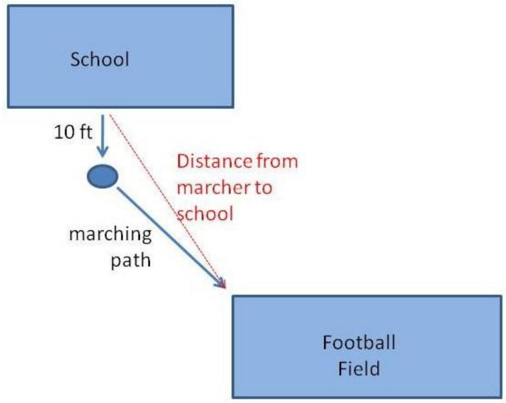


Figure 7. One possible situation model for marching band problem given to Amy

Another issue Amy had with this problem seen elsewhere in this pilot study related to the four-part problem structure, where the student is first asked to solve two results unknowns ("concrete cases"), then is asked to write a symbolic expression, and then finally is asked to solve a start unknown. Each part of this problem refers to the passage of time, using the following sequence:

a. How far will you be from the school in 30 more seconds?
b. How far will you be from the school in 2 more minutes?
c. Write an algebraic expression for distance from the school as a function of time.
d. In how many more seconds will you reach the football field, which is a total of 600 feet from the school?

When Amy reached part d), she asked the interviewer if she should consider the "movement" that had occurred in part b), i.e., was she now starting her march 360 ft from the school instead of 10 ft, since in part b) she had marched that far? This seemed like a reasonable assumption to make, however it would have led Amy down an unintended solution path. Although at this point in the study the hypothesis was that personalized problems would scaffold problem solving, here such concerns seemed to be completely overruled by issues with problem wording and structure.

For part c) of the marching problem where she was asked to write a symbolic expression, Amy seemed to initially freeze up, telling the interviewer "I don't get this one." When the interviewer clarified, Amy first wrote "3 ft x" erased it, and then wrote simply “3x” explaining that “you are multiplying it by however many seconds.” Amy did not add on the needed intercept value of 10 feet to her symbolic equation. Amy was given two more story problems that contained intercept terms in the problem framing, and also did not include an intercept term in the symbolic expression she wrote for either of these. However, she used the intercept terms when solving the associated result unknown problem parts. This seems to suggest that the formal mathematical objects that Amy is representing with symbols are disconnected from the situation-based reasoning she uses to solve the result unknowns.

Amy struggled not only with including the intercept in her symbolic expressions, but with remembering to take the intercept into account when solving start unknowns. The marching problem was the only problem in which she remembered to take into account an intercept in part d), when she suddenly realized, “Are you doing like, is it saying minus the 10 because you’ve already gone 10 feet?” Overall, Amy was able to solve result unknowns relatively easily after her early struggle with the marching problem, but continued to have difficulty compensating for an additive start value when solving start unknowns and writing symbolic expressions.

B. Interviews with Mark

Mark was also a 9^{th} grade Hispanic student enrolled in Mrs. C’s Algebra I class. During the entrance interview, he talked about how he enjoys playing basketball in his neighborhood, and one way he uses numbers is by adding up the points as he plays. He discussed how he normally uses math to add up his totals and calculate tax when shopping, and that learning math is important so that you don’t get “ripped off.” He also discussed how he was taking dance lessons to prepare for a Quinceanera, and his visits to see family in Mexico during the summer. Based on the entrance interview, two personalized problems were written for Mark; in one Mark was accumulating points while playing basketball, and in the other he was taking a road trip to Mexico to visit his grandparents.

During the problem-solving interview, Mark worked confidently through his four problems, but like Amy encountered issues interpreting the intercept in this style of story problem. For his first problem, Mark was given the normal version of problem 1 in Table 1: “A machine called the Crawler, which moves space shuttles, travels at the rate of 3 feet per second. The Crawler is currently 10 feet from the hangar, moving toward the launching pad.” Mark repeatedly referred to the Crawler’s movement as the machine “rising up,” like the Crawler was the space shuttle that was flying, so it was unclear whether he conceptualized the situation model in the way the authors intended, or understood the meaning of the terms *hangar* and *launching pad*. The series of questions posed with this problem were:

a. How far will the Crawler be from the hanger in 30 more seconds?
b. How far will the Crawler be from the hanger in two more minutes?
c. Write an algebraic expression for the total distance from the hangar as a function of time.
d. In how many more seconds will the Crawler reach the launching pad, which is a total of 600 feet from the hangar?

Mark solved part a) without issue, and when he got to b), instead of directly using the 2 minutes (120 seconds) in his calculation, he used 90 seconds and then added his answer to his response from part a). This resulted in the correct answer of 370 ft, and showed that Mark, unlike Amy, understood that the 30 seconds had not already passed in part b). In part c) Mark wrote the algebraic expression as “D = 3s,” like Amy leaving out the intercept term when symbolizing. However, in this problem and in Amy’s problem the intercept term seems to be without strong situational meaning. Why should the Crawler’s 10 foot distance from the hanger at some arbitrary moment when this problem was posed be significant?

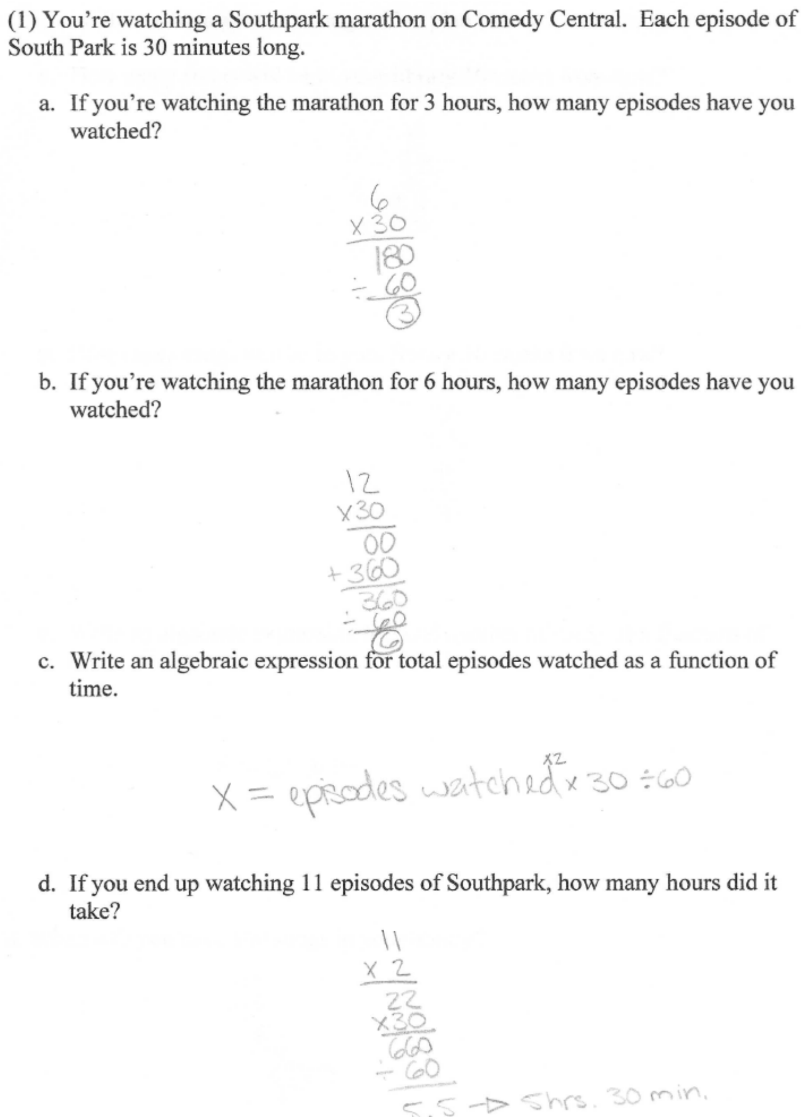
In part d), Mark’s response was again interesting – he seemed to apply the word “total” incorrectly as a keyword, saying “So it says 600 feet total, it’s asking from the previous question, so I guess I subtract [370] to find out how many more it’s going to rise.” Here Mark had the same type of misunderstanding that Amy initially had with her marching problem, and he took into account “movement” occurring in part b) when solving part d). The word “total” seemed to cue Mark to use this approach, as it was accentuated in his speech when he was explaining his reasoning. Mark ran into similar issues with his personalized basketball problem; he took into account movement that had occurred in previous parts, and left out the intercept term when symbolizing.

Although over half of Mark’s responses to these two questions were unintended, given the ambiguous problem wording and lack of significance of the intercept term in the story context, it is clearly not the case that Mark was suspending his sense-making of the situations being presented. Further, although the basketball problem Mark solved later was a personalized scenario, issues with problem semantics overrode any benefit Mark could have gained through his familiarity with accumulating points in basketball.

C. *Interviews with Carl*

Carl was a 9th grade Hispanic student enrolled in Mrs. C’s Algebra I class. During his entrance interview, he discussed how he enjoys playing video games, watching TV, playing sports, and listening to music. He also mentioned how he likes to use the computers at his community center after school, and that he downloads songs from iTunes. Two personalized problems were written for Carl based on the entrance interview; in one problem Carl was watching a Southpark marathon on Comedy Central, and in the other he was downloading songs on iTunes.

In the problem-solving interview Carl worked through his four problems efficiently, and except for one problem got each part of each problem correct and was able to clearly explain his reasoning when prompted. Carl’s work on his first personalized problem is shown in Figure 8; this was the only problem he had difficulty with. In parts a) and b), Carl seemed to know immediately that he would watch 6 episodes in 3 hours and 12 episodes in 6 hours: “Okay oh, well each episode is thirty minutes, so there’s three hours and you’ve watched like 6 episodes and 12 episodes in 6 hours.”



(1) You're watching a Southpark marathon on Comedy Central. Each episode of South Park is 30 minutes long.

a. If you're watching the marathon for 3 hours, how many episodes have you watched?

$6 \times 30 = 180$, $180 \div 60 = 3$

b. If you're watching the marathon for 6 hours, how many episodes have you watched?

12×30: $00 + 360 = 360$, $360 \div 60 = 6$

c. Write an algebraic expression for total episodes watched as a function of time.

X = episodes watched x2 x 30 ÷ 60

d. If you end up watching 11 episodes of Southpark, how many hours did it take?

$11 \times 2 = 22$, $22 \times 30 = 660$, $660 \div 60$

5.5 → 5hrs. 30 min.

Figure 8. Carl's work on his first personalized problem

However, Carl seemed to then become confused over whether the question was asking for "hours" or "episodes," and continued to work on answering part a): "Because 6 times 30 is…zero times 6 is zero and 3 times 6 is 18 and then you divide that by 60 and that gives you three. So that's 3 hours because there is sixty minutes in an hour." Rather than recognizing his initial calculations as the answer the problem was asking for, he used these values to go backwards and derive the "x" values that had been provided in the problem. Carl's answer to part c) further suggests that he seems to be interpreting number of episodes watched as the independent quantity, and he verbally names as his dependent variable x as “hours there are that you’re watching the marathon.” Carl’s symbolic expression in part c) would have been correct if he had not added in the “x 2” as an afterthought.

Carl seemed to informally understand this idea of doubling to covert between hours and episodes, which was perhaps a result of this problem actually relating to his everyday experience of watching repeated, 30-minute TV shows. Mark, the previous student, had been given a generic version of this same problem where he was completing “30 minute tasks,” and did not show any explicit understanding of this idea of doubling. However, Mark solved each part of his generic problem correctly, writing the symbolic expression as $t = h \div 30$, while Carl’s attempt to use his situational knowledge may have actually disrupted his problem solving.

Carl’s response to d) was equally interesting – it seemed that he was trying to use the equation he generated in part c), but when he gets to the final step of $660 \div 60$, instead of putting 11 as his answer, he writes 5.5 or 5 hours and 30 minutes. It could be that Carl informally understood the final question, and decided to rely on his situational knowledge rather than the calculational answer generated by his process in part c). Here his decision to follow his informal knowledge rather than his formal equation resulted in the intended answer.

At the end of the interview when Carl was asked which problem was easiest, he named the Southpark problem, saying “I watch 30 minute long shows on TV so I pretty much know how much shows are in each hour. So yeah, it makes it easier.” The previous student Mark had named the generic version of this problem on 30-minute tasks as his most difficult problem. When Carl was asked which problem was most difficult, he responded that the normal version of problem 2 in Table 1 was most difficult, because “in the word problem there was a lot of extra information that I did not need to use.” In terms of mathematical structure, this problem was easiest because it did not have an intercept, and Carl had solved all parts of this problem correctly. The first language spoken in Carl’s home is Spanish, and it is interesting that he chose the one problem that he got wrong as the easiest, and the problem that he solved most easily as the hardest, apparently based on how he felt about their "cover stories."

D. Interviews with Matt

Matt was a 9th grade Caucasian student enrolled in Mrs. C's Algebra I class. While the other four students who participated in the pilot study were identified by the teacher as generally being high achievers, Mrs. C identified Matt as struggling with mathematics. Matt gave one of the most interesting responses when asked why it was important to learn math, saying "Well math is… basically everything revolves around math, I mean, there is not one job where you don't have to use some form of math in. So it's really good to learn math and everything to be able to survive out there in the world."

During the entrance interview, Matt talked about how he likes to play sports like football and wrestling. He further described how he likes to play role-playing games so that he can "live another life," with one of his favorite online RPGs being Runescape. Based on the entrance interview, three personalized problems were written for Matt; in one problem he was accumulating experience points in Runescape, in the second he was downloading music on iTunes, and in the third he was watching a Southpark marathon on Comedy Central (same problem as Carl).

During the problem-solving interview, Matt was very talkative, but also seemed nervous and regularly asked for the interview's input. For each problem Matt's biggest struggle was with writing a general algebraic expression; he seemed very intimidated by symbolism. Matt first solved the Southpark problem (same problem as in Figure 8), using his informal knowledge to answer parts a) and b) quickly and accurately. However, when he reached part c), Matt seemed to almost panic saying:

> I never quite got these; Mrs. C has always been there to help me. But an algebraic expression… I'm guessing that, I don't know what numbers would be the x, the y, or anything so… time always goes on the x-axis, so three… does the x come before the y in the algebraic expression?

Matt abandoned this part of the problem in order to successfully solve d), and when he came back to it he again made it clear that he did not understand how to write an expression. Matt was able to get 3 out of 4 parts of this problem correct by relying on his informal knowledge, while Carl, a much stronger student, failed at several parts of this problem by attempting to apply more complicated algorithms.

Matt's proposition that "time always goes on the x-axis" is also of note, given that in this problem and others, time can be a dependent quantity. Matt seemed to use this type of non-coordinative or direct translation approach at several points when his informal knowledge failed. For example, when given his next problem statement "Your iTunes library has 11 songs. You plan to buy 2 more songs every week," his first utterance was "Your iTunes library has 11 songs, so that means if I have to write an algebraic expression, I'm always going to have to put plus 11 at the end." However, this strategy did not translate to success in part c) of this problem, where he wrote the algebraic expression as "y2 + xw + 11" (the w stood for weeks).

Throughout the interview, Matt seemed to be familiar with how to place intercept terms in symbolic expressions, but had difficulty conceptualizing a relationship between two variables, differentiating between independent and dependent quantities, and representing rate of change. However, this did not stop him from using sophisticated and efficient arithmetic techniques to solve parts a), b) and d) of each problem successfully. When he was solving these parts he seemed to understand a variable as a set, and its relationship to rate of change. However when he approached part c) where he was asked to symbolize, a variable seemed to become a fixed, unknown entity that made little practical sense.

E. Interviews with Lisa

Lisa was a 9^{th} grade Hispanic student enrolled in Mrs. C's Algebra I class. When asked why she thought it was important to learn mathematics during the entrance interview, she responded jokingly "Because you're going to need it all through your life. And if you go without it you'll be stupid basically." She described how she used mathematics in her everyday life to figure out prices when shopping, but could not come up with another everyday use for mathematics when prompted.

During the entrance interview, she discussed how important her social life is, and how she hangs out with her friends and is “rarely home.” She described how she is on the computer a lot using MySpace, instant messaging, and downloading music on iTunes. Lisa also talked about how she loves watching movies and going shopping for clothes and accessories with her friends. Two personalized problems were written for Lisa based on the entrance interview; in one problem Lisa is saving money to buy clothes from her favorite store, American Eagle, and in the other she is adding friends on MySpace.

During her problem-solving interview, Lisa was successful at solving each problem she was given, although she was constantly making comments about how she was sure she was getting them all wrong. The problem she struggled most with was the normal version of problem 4 in Table 1, reading:

> A company has been created to produce a new product. The company predicts that its capital expenditure (the one-time start up costs to buy supplies, equipment etc.) will be $500. It plans to sell its new product for a profit of $10 per unit. (The profit per unit is the price at which it sells each unit minus the costs to make and sell each unit.) The company's profits for its first year of operation will be its total profits from sales minus its capital expenditure.

This scenario was chosen for the pilot in part because of its large amount of text and high level vocabulary. The problem had four parts:

a. If the company sells 190 units during the first year, how much total profit will the company make?
b. How much total profit will the company make if it sells one hundred and fifty units?
c. Write an algebraic expression for profit as a function of the number of units sold.
d. How many units will the company have to sell to break even?

Lisa worked through parts a) through c) without taking into account the intercept term of -500. When encouraged by the interviewer to go back and read the problem more carefully, she corrected her answers after some thought. When she reached part d), she became confused and did not know how to answer the question. When the interviewer asked her to describe what “break even” meant, she said “Like an even number is what I think it means, but I don’t understand what the even number they’re trying to get is.” Amy, the first student whose interview was discussed in this section, was given a personalized version of this same problem and also told the interviewer that she did not understand what “break even” meant. Amy’s personalized version of this problem contained a reduced problem text and presented a simple scenario where Amy was planning a band fundraiser party, and it is worth noting that unlike Lisa, Amy had no issues with remembering to take into account the -500 intercept. However, both students had difficulty with the final part of this question due to issues with vocabulary interpretation.

When asked which problem was easiest and which problem was hardest, Lisa, unlike Carl, seemed to be attuned to the mathematical structure of the problems. She chose the problem that had the simplest mathematical structure (no intercept) as easiest, even though this was the same problem Carl had chosen as hardest. She chose the problem with the most sophisticated mathematical structure (the problem with the company sales) as the most difficult.

F. Discussion

Reflecting on the second pilot study after it was completed, a number of lessons were learned that would be used to design and execute the dissertation study. First, it was observed that there were two distinct types of personalized problems being presented to students. The first type of personalized problem related to student interests, but was not connected to the ways in which the students actually used mathematics in their everyday lives. An example of such a problem was “You are marching at band practice at a rate of 3 feet per second. You are 10 feet away from the school, marching towards the football field. How far will you be from the school in 30 more seconds?" While Amy was certainly interested in her role as a member of her school band, it is unlikely that she thinks about marching in terms of rate of change, or that there would be any specific use to thinking about marching in this way.

The other type of personalized problem used during the pilot study was also related to students' lives and interests, but was more likely to connect to students' participation practices in everyday situations. An example of such a problem was "You are watching a South Park marathon on Comedy Central. Each episode of South Park is 30 minutes long. How many episodes can you watch in 2 hours?" This story seemed to be a closer representation of how students might actually use and understand quantitative relationships in their day-to-day activities. As a result of this observation, in the dissertation study the focus was on the second type of personalized problems – problems designed to connect to the ways in which students may conceptualize quantitative relationships in their lives. As a result of this changed focus, the entrance interview questions were modified to be more targeted to this goal.

Second, it was decided that the interviewer needed to take a larger role in questioning students during the problem-solving interview, prompting them to explain their thinking, and reactively asking for specific explanations when something of interest occurred. Third, this pilot study showed the importance of the symbolic representations that students were being asked to construct in part c) of each problem, and how these representations were tied inexorably to students' understanding of the problem structure, their grasp of central ideas in algebra, and their modes of participation in school mathematics. Thus it was determined that more attention would be paid to questioning students about their understanding of symbolic representations.

Finally, the second pilot study demonstrated that notions of situation facilitation and personalization in traditional story problems were in many ways overshadowed by other elements of the system of school mathematics. Certainly, there were some cases where wording and vocabulary seemed to make a difference, but there were also instances where students seemed to be making reasonable inferences about a problem that were incorrect with respect to the intended solution path. The second pilot brought to the forefront the ways in which traditional word problems artificially constrain problem solving and fail to take into account the diversity of understandings and approaches that students bring with them to problem solving, and this became an important focus for the dissertation study.

G. Revised Model of Story Problem Solving

Based on the literature review and the findings of the pilot studies, it was useful to return to Greer's model of story problem solving, and consider a more systematic view of how sociomathematical norms interact with problem-solving actions. Figure 9 shows a modified version of Greer's original model of story problem solving (which was shown in Figure 5).

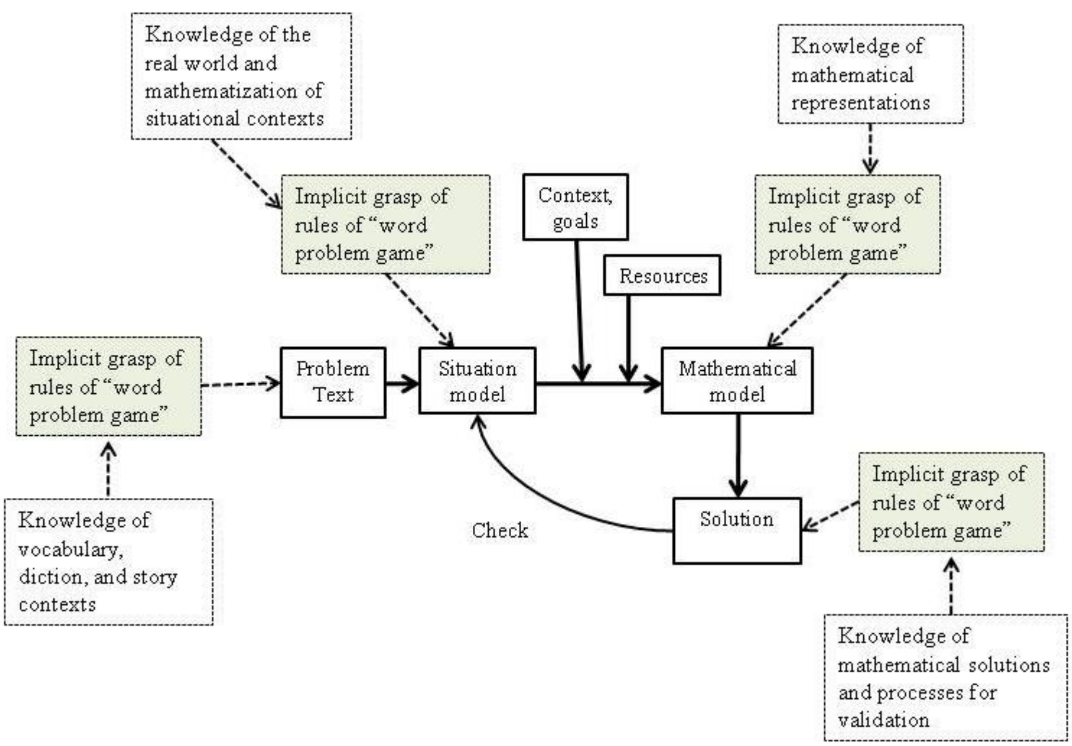


Figure 9. Revised model of problem solving, based on the two pilot studies and literature review (see Figure 5 for original Greer (1997) model; solid lines show original Greer model)

In Figure 9, rather than the rules and norms of the “story problem game” affecting only one phase of problem solving, the formation of the situation model, in the new model they permeate all aspects of problem solving, from the interpretation of the textbase, to the formation of the situation model, to the ways in which students use and understand mathematical representations while developing a more formal mathematical model of the problem, to the decisions students make about whether to accept or reject a possible solution. This model is more consistent with the literature on story problem solving, and also is supported by the findings of the second pilot study.

V. Chapter Five: Problem Statement

In the first pilot study, video observations of a high school algebra class learning to use the Cognitive Tutor Algebra software were conducted. Transcripts showed the students and the teacher working to adapt to a system of sociomathematical norms envisioned by the creators of the software program for the learning of mathematics. Many of these norms surrounded the format, structure, and expected strategies for the story problems presented by the curriculum.

In the second pilot study, five students were given standard story problems from the Cognitive Tutor Algebra curriculum to work on paper, as well as versions of these problems that were personalized to their interests or rewritten with simplified language and general referents. Analyses showed that the idea that students can use participation practices from everyday situations to support them when solving story problems may be problematic, even when problems are personalized. Also, there were more examples of students struggling to adapt to the norms and expectations for solving story problems and creating symbolic representations. Finally, students chose to use informal strategies rather than symbol manipulations.

The remainder of this paper reports on the dissertation study that these pilot studies led to. The goal of the dissertation study was to extend the analysis of the sociomathematical norms surrounding the solving of story problems, using these norms to critically examine two of the common justifications for teaching mathematics in context – providing access to mathematical ideas and allowing school-learned mathematics to transfer to out-of-school situations. The dissertation study was designed to have a larger sample size, in order to allow for some discussion about the prevalence of different problem-solving issues and behaviors, as well as to allow for saturation of problem-solving constructs of interest. The dissertation study also was designed to have more types of problems, in order to begin to systematically analyze how problem framing and context impacts problem-solving behaviors.

Research results relating to use of everyday participation practices, impact of personalization, issues with verbal reasoning, and effects of the system of schooling have not been examined for story problems at the algebra level, and these issues have important implications relating to the justifications for teaching mathematics in context. The overarching aim was to engage in an exploratory interview study where students not traditionally successful at participating in school mathematics were presented with algebra story problems to solve. The research purposes were threefold:

1) To examine personalization of story contexts to investigate how contextualizing mathematics with respect to students' everyday experiences affects problem solving in algebra.
2) To analyze the common justifications for putting mathematics in context, such as providing accessibility through verbal and situation facilitation, and the affordances and constraints of story problems for these purposes.
3) To study students' informal and formal strategies to solve algebra story problems, their use and understanding of symbolic representations, and how these practices are imbedded in the larger system of school mathematics. Also examined is how use of these practices and tools varies by problem context and framing.

With respect to the first purpose, there was expected to be a benefit for personalizing algebra problems to individual students' experiences. These problems should be more accessible and easier to solve, and students should be better able to draw on their everyday knowledge and experiences to form a more elaborated understanding of the situation, or situation model. With respect to the second research purpose, it was conjectured that story problems would show some of the assumed benefits for placing mathematics in context, but would also have important shortcomings that have been underemphasized in the literature. With respect to the third purpose, previous literature suggests that students would use a variety of informal strategies to solve problems, and would rely on these informal strategies more than formal approaches like equation solving. Many students may be caught in arithmetic rather than algebraic modes of thinking, but a key goal of this study was detailing the "funds of knowledge" (Moll & Gonzalez, 1997) that students bring with them to algebra class.

Theoretically, these studies were framed with respect to the situated cognition perspective (Greeno, 2006). From this perspective, when students solve story problems, they are participating in the complex, social system of "school mathematics" that has its own norms, practices, and standards of performance, including norms relating to the use of representational systems like symbolism. This framework is powerful because it provides explanatory power for research findings that participation in school mathematics does not always enhance participation in applied problem solving in other systems.

The intended theoretical contribution was twofold. First, the dissertation study sought to detail the sociomathematical norms and practices surrounding participating in school mathematics by solving story problems. Calling attention to norms and practices that correspond *only* to the system of schooling would show that story problems may be an impoverished conceptualization of "contextualized mathematics." This has been a neglected area of research for algebra story problems, where the norms surrounding the use of abstract representational systems become central, and "everyday" use of concepts becomes less common.

Second, the dissertation study sought to determine whether presenting story problems in different formats, such as personalized to the students' interests, would change the way students thought about the task, and the approaches they used to solve problems. When students are participating in a school mathematics activity, it is clearly possible that their modes of participation will enhance or be related to their participation in other systems, or more generally will enhance their conceptual understanding of mathematical ideas. Thus an important goal was to gain an understanding of what types of contextualized problems afford these opportunities, as well as what problem framings allow access to students who struggle to adopt the participation practices of school mathematics.

Overall, the dissertation study sought to form a theoretical explanation for why different problem framings or types of contextualization may cause students to reason differently, in the case of story problems. In the model presented in the previous section, differences in students' responses to story problems were in part attributed to their "implicit grasp of the rules of the "word problem game.'" However this model does not explain why a single student may choose to reason differently when presented with different types of story problems, or use everyday participation practices when solving some story problems but not others. The idea of knowledge being filtered *only* through students' grasp of the sociomathematical norms surrounding story problems did not seem like an adequate way to fully model story problem solving, so the dissertation study sought to develop a framework with increased explanatory power.

VI. Chapter Six: Methodology

A. Participants

A series of semi-structured problem-solving interviews of algebra students from a high school in a diverse urban district in Texas were conducted. Data collection took place over the 2008-2009 school year, with the 5 aforementioned pilot students being interviewed in Fall 2008 (November/December), and 19 additional students being interviewed in Spring 2009 (April/May). Seventy-four student participants were recruited for the study from three different Algebra I classes of the same teacher, who is referred to as “Mrs. C.” Students were asked during class by the primary researcher if they would be willing to participate in an interview where they would solve algebra problems while being audio-recorded for a small stipend. Parental consent was obtained from 39 students (52.7%). Due to issues with time constraints, student mobility, and student absenteeism, 29 of the 39 students participated in an entrance interview, and 24 of these 29 students participated in a problem-solving interview where they were given algebra problems, conducted on a different day.

Of the 24 students that participated in this problem-solving interview, 13 (54%) were Hispanic, 8 (33%) were White, and 3 (13%) were African-American; this distribution is close to the school distribution given shortly. Of the 24 students, 14 (58%) were male, and 10 (42%) were female, compared to 53% male and 47% female at the school. For five (21%) of the students in the study, the primary language spoken by the student’s guardian who received the parental consent form was Spanish. Nineteen (79%) of the 24 students were eligible for free or reduced lunch, compared to 75% of all ninth grade students; this is used as an indicator of low socio-economic status. Fifteen of the 24 participants (62.5%) passed the state standardized mathematics exam in the year of the study, compared to 62% of all ninth grade students at the school. Nineteen of the 24 participants (79.2%) passed the state standardized reading exam, compared to 80% of all ninth grade students at the school. An analysis of participant standardized test scores is presented in the final chapter.

Students from this demographic background were chosen purposefully based on the research questions. It was of interest to study a population that was not traditionally successful within the system of school mathematics, where there would be a tension between what students know and understand and what the system of schooling asks of them. By studying this population, there was the most potential to show discontinuities between in-school and out-of-school participation practices for students who had not become so adept at "playing the game" of school mathematics that these discontinuities had become trivial to achievement. Further, it was important to work with a population that had a greater diversity in language, cultural, and social participation schemes, since these are important to school achievement (Au, 1980; Cobb & Hodge, 2002; Heath, 1982; Khisty, 1995; Ladsen-Billings, 1995; Moll & Gonzalez, 1997; Nasir, Roseberry, Warren, & Lee, 2006).

B. School and Classroom Contexts

The main researcher maintained a presence in Mrs. C's classroom on a regular basis throughout the 08-09 school year as a participant-observer in order to better understand the students' school mathematics context. A description of the classroom and school environment, the curriculum, and Mrs. C's instruction and assessment approaches is provided in this section. In qualitative research, issues of generalizability are sometimes framed instead as transferability, where readers use rich, thick descriptions provided by researchers to determine the degree to which the results of the study would transfer to a setting with which they are familiar (Anfara, Brown, & Mangione, 2002; Merriam, 2002). Thus the prolonged engagement in the classroom here allows for such thick descriptions of the school and classroom settings.

Maintaining a presence at the school throughout the year was also essential because when conducting interview research, familiarity with context is important in order to gain an understanding of the local language and references interviewees make (Kvale, 1996). Finally, in order to gain a greater understanding of how these students solved problems, the main researcher often graded student work during Mrs. C's conference period and lunch period.

The high school where this study took place was located in a large, urban district in Texas. The school's student population was 65% Hispanic, 22% White, 11% African-American, and 1% Asian/Pacific Islander, with almost 2000 total students. The student population was 58% economically disadvantaged, 13% Limited English Proficient, and 74% "At Risk[1]." The year prior to the study, the school had been rated "Academically Unacceptable" in mathematics under the guidelines of *No Child Left Behind*, with only 51% of students in the 9th grade passing the state standardized mathematics exam. This was not the school's first unacceptable rating, and the school was under pressure to improve mathematics scores. During the year of the study, the school also did not make Adequate Yearly Progress (AYP) under the guidelines of *No Child Left Behind*, due to the mathematics scores of two student subgroups. Over the course of the research conducted at this school, there were 3 different principals. The principal during the year of the study was strongly focused on maintaining student discipline and improving standardized test scores.

Ninth grade students at the school were typically enrolled in Algebra I, which met for 1 hour and 40 minutes on alternating days. Students taking Algebra I were also required to take a Math Lab course, which provided focused instruction and practice of 8th grade state mathematics standards tested on the 9th grade state assessment. The Math Lab class met for 1 hour and 40 minutes on the other days. The district had previously been using a curriculum that integrated some reform-based ideas, and Math Lab had originally been framed as a project-based applications course. However due to federal and state accountability pressures, a committee of teachers selected a "back-to-the-basics" textbook series for adoption in 2006, and Math Lab increasingly became used for below grade level standards review and standardized test preparation.

[1] A student being classified as "At Risk" is based on a number of factors, including their grades, standardized test scores, and whether they have been retained. See http://ritter.tea.state.tx.us/peims/standards/wedspre/index.html?e0919 for a full definition.

Students participating in this study were recruited from the classes of one teacher, Mrs. C. As previously mentioned, Mrs. C was in her fourth year of teaching algebra at the school site. Before working at the school, she had obtained a degree in mathematics and been certified as a secondary mathematics teacher through a university program. The Algebra I and Math Lab classes of Mrs. C usually consisted of short lectures and note-taking, followed by students completing worksheets at their desks. On most days there would be between 1 and 3 of these lecture-worksheet cycles during the class period. The worksheets contained short, closed-ended problems often targeted to standardized test preparation, and often framed as traditional story problems.

Mrs. C confirmed the prevalence of story problems in her classroom; when asked how often she used story problems in her teaching, she responded “all the time” (Mrs. C, May 15, 2009). Most of the materials, assessments, and worksheets used by Mrs. C were used by all algebra teachers at the school; the math department collaborated on their class planning. Occasionally a more extended problem-solving investigation or activity, often from the previous district curriculum, would make it into the day’s lesson sequence. However, when this happened, the activity would often be broken down step-by-step, sometimes through explicit additions made prior to instruction, and thus these activities ended up being similar to the regular worksheets.

Students would sometimes work together to complete their worksheets, but most often students would either work individually, not complete the worksheet, or copy another student’s worksheet in the final minutes of class. Textbooks were not used during class, and although some students took notes when the teacher lectured, these students almost never referred to their notes when completing a worksheet. When students did not know how to immediately work one of the problems given to them, they would most often raise their hand and get help from the teacher, skip the problem or stop working, or copy the answer or get help from another student.

The grading system for Algebra I classes at the school was also standardized. Mrs. C explained that their grading system took after sports – you only get credit for the games, not the practices. Thus the students’ grades were largely (around 90%)

determined by exams, particularly the district-wide benchmark exams for mathematics given every six weeks that were intended to assess students' progress towards being able to pass the state standardized exam. The algebra department at the school closely monitored the students' performance on benchmark exams and compared these scores to the NCLB requirement that all ninth graders and all student subgroups reach 55% passing in the year of the study. As the state standardized test approached, students were grouped by the state math standards they were weak on, in order to receive targeted instruction on those topics. Graphing calculators were not used often in class until the standardized test was near. Many of the calculators were vandalized to the point where they were barely usable, and there were not enough for all the students in the class.

The community in which 19 of the 24 students (79%) lived had a median household income of $54,000 and an average home value of $91,000. The community was 45% Hispanic with a median age of 31 and an average household size of 2.55 people. Approximately 10.7% of the community's population did not have a high school diploma or GED, while 24.6% of the population had an Associate's degree or higher.

C. Data Collection

Each student first participated in a 10-15 minute a semi-structured entrance interview face-to-face that was audio recorded. The interviewer asked a series of questions related to how the student used math in their everyday life, where they see and have to deal with numbers, what types of activities and hobbies they are interested in, and what they believe is the purpose of learning algebra in high school (Appendix A).

After the entrance interview had been conducted, a set of five algebra problems on linear functions was written for each student. Two of the problems were personalized according to how the student described using mathematics and numbers in their everyday life, while the other problems were either typical story problems on linear functions from the Cognitive Tutor Algebra curriculum, generic versions of these problems with simplified language and general referents, or completely abstract symbolic equations. Some of the typical story problems included a symbolic equation in their text to express the story's relationships. See Table 3 for examples of all the different problem types given to students.

Problem Type	Example
Normal Story Problem	Some early Native Americans used clam shells called Wampum as a form of currency. Tagawininto, a Native American, had 80 wampum shells, and spends 6 of them every day. a) How many shells did Tagawininto have after 10 days? b) How many shells did he have after a week? c) Write an algebra rule that represents this situation using symbols. d) After how many days did he have 8 shells?
Story Problem with Equation	Some early Native Americans used clam shells called Wampum as a form of currency. Tagawininto, a Native American, has a number of wampum shells given by y=80-6x, where x is the number of days that have passed. a) How many shells did Tagawininto have after 10 days? b) How many shells did he have after a week? c) After how many days did he have 8 shells? d) What does the 80 represent in this situation? What does the 6 represent?
Personalized Story Problem	You are playing your favorite war game on the Xbox 360. When you started playing today, there were 80 enemies left in the locust horde. You kill an average of 6 enemies every minute. a) How many enemies are left after 10 minutes? b) How many enemies are left after 7 minutes? c) Write an algebra rule that represents this situation using symbols. d) If there are only 8 enemies left, how long have you been playing today?
Generic Story Problem	You have 80 objects, and lose 6 every day. a) How many objects will you have after 10 days? b) How many objects will you have in a week? c) Write an algebra rule that represents this situation using symbols. d) After how many days will you have 8 objects?
Abstract problem	$y = 80 – 6x$ a) If $x=10$, what is y? b) If $x = 7$, what is y? c) If $y =8$, what is x? d) Write a story that could go along with the equation $y = 80-6x$.

Table 3. Problem types given to students during the problem-solving interview in the full study

In the first two parts of each problem, the student was asked to solve for y given a specific x-value – these are typically called *result unknowns* (Koedinger & Nathan, 2004). The student was then asked to write an algebra rule representing the story, and finally was asked to solve for x given a specific y-value, typically called a *start unknown*.

In the two problem types where the student was provided the symbolic equation (story with equation and abstract problem types) instead of being asked to write an algebra rule, the student was asked to either interpret the parameters (slope and intercept) of the given equation in the context of the story scenario, or write a story to go along with the given equation.

All of the problems used were variations of 13 base story problems pulled from the Cognitive Tutor Algebra curriculum. The 13 base scenarios were chosen so that when students mentioned a numerical relationship during the entrance interview, there would be a variety of number choices (decimals, whole numbers, large numbers, small numbers, positive numbers, negative numbers, percents) and problem structures (positive slope no intercept, positive slope positive intercept, negative slope positive intercept, etc.) available to appropriately personalize a problem to their experiences. See Appendix B for a list of the 13 base problems used in this study. The order in which problems of various types (personalized vs. normal vs. abstract etc.) were presented to students was randomized.

D. Interview Methodology

The interview methodology used for the problem-solving interviews was again a semi-structured interview. Merriam (2002) offers the following definition, which fits well with how the interviews in this study were conducted:

> A semi-structured interview contains a mix of more and less structured questions. Usually, specific information is desired from all participants; this forms the highly structured section of the interview. The largest part of the interview is guided by a list of questions or issues to be explored, and neither the exact wording nor the order of the questions is determined ahead of time. (p. 13)

Each student had five problems to solve, with each question written in advance and matched in terms of mathematical structure with the problems other students would be solving. Students were asked to think out loud as they solved their problems, and were explicitly instructed to construct self-explanations (Chi & VanLehn, 1991). As students construct self-explanations, they must think about verbalizing their processes in a way that would make sense to a third party, namely the interviewer, thus a self-explanation methodology is not the same as a think-aloud protocol (Ericsson & Simon, 1998). Allowing the students to think aloud uninterrupted would not have allowed the research questions to be addressed; it was necessary for the intercviewer to understand why the student was performing actions and the meaning they ascribed to them in order to analyze how systems of norms interacted with their problem solving.

At the beginning of each interview, the researcher would tell the student that they were going to solve some problems today while thinking aloud and explaining each of their steps. Each interview began with the student solving a multiplication problem with two multi-digit numbers as a warm-up, so they would have practice thinking aloud. The interviewer would then tell the student that they would be audio-recorded solving five more problems. The interviewer let the student know these problems were not for a grade, but to try their hardest to solve each one. The interviewer also instructed students that they could not ask for assistance from the interviewer on any of the problems. The interviewer gave each student a calculator to use to solve the problems, but told them if they were going to use it, they needed to say out loud each button they pushed. Calculators had not been provided in the second pilot study, and as a result many students had spent lengthy periods doing by-hand calculations.

The student would then begin working through the problem set, showing work on a blank sheet of paper with the problem printed on it, and verbalizing each step. When the student forgot to verbalize steps, or their thinking was not clear, the interviewer would give a standard probe asking the student what they were doing. The interviewer also used spontaneous questioning based on what was occurring in the interview. For example, if the student's work and verbalizations made the interviewer suspect that they had misinterpreted the problem, the interviewer may ask the student to define words in the problem situation, or to describe what was going on in the problem situation.

When conducting such qualitative interviews, the discourse produced is a joint construction of interviewee and interviewer, where each partner influences and enacts change in the other as they build a joint understanding of the topic under consideration (Kvale, 1996). In this study, the interviewer asked the students probing questions during the interview as well as pre-determined direct questions (detailed in the next section) about the meaning and purpose of their actions. This methodology is similar to the Neuman and Schwarz (2000) and Clement (1982) studies on algebra problem solving. From this perspective on interview research, analysis of interview data must be accompanied an acknowledgement of the role of the interviewer as a coauthor of the discourse (Kvale, 1996).

The social and emotional atmosphere during the interviews varied; the students were all nervous to some degree. Indeed, even college-educated adults would likely be nervous if asked to solve algebra word problems without preparation or assistance. However, students were familiar with and had a reasonable level of rapport with the main researcher due to their presence throughout the year in the classroom, and all students without exception were cooperative in terms of their willingness to solve the problems and explain their thinking. The main researcher purposefully waited until the students were accustomed to her being in the classroom on a regular basis over several months before beginning the problem-solving interviews; this was a population of students in which many had negative and stressful experiences with mathematics.

Each interview closed with several pre-determined questions where students were encouraged to reflect over the problems they had just solved. Students were asked to pick the easiest and hardest problems they had solved, and give justifications for their choices. Students were also asked what they thought was the purpose of being given story problems to solve in algebra class.

An interview with the teacher in the classroom, Mrs. C, was also conducted, and the questions posed during this interview are provided in Appendix C. The teacher was asked about her beliefs on story problems and her classroom use of story problems, how she used math in her everyday life, how she thought students used math in their everyday lives, and what she thought was the purpose of learning algebra. She was also asked to solve the two most difficult problems posed to students during the interviews, and to discuss how successful her students would be at solving these problems and explain why.

E. Designing "Relevant" Algebra Story Problems

During each entrance interview, students were asked how they use mathematics in their everyday life; responses are shown in Figure 10 (left). The most common response (14/30 responses) was the student describing how they use math when shopping to add prices or count money, which are typical arithmetic scenarios. The second most common response (7/30 responses) was the student saying that they did not use math apart from schoolwork. Three students mentioned using math when measuring ingredients in cooking, while another 3 students described using simple arithmetic computations at their work, such as determining the amount of change to give a customer.

Since many students seemed intimidated by the word "math," students were also asked as a follow-up question where they see and have to deal with numbers in their everyday lives; responses are shown in Figure 10 (right). These responses were somewhat different, however the most common response (6/32 responses) was the student saying they do not use numbers outside of schoolwork. An equally common response (6/32 responses) was the student describing using numbers with technology; for example, one student said that they use numbers "When I'm downloading music from the computer, or when I'm putting software on the computer." Three additional categories also got 4 responses each – students once again mentioning using numbers at the store, students mentioning seeing numbers on bills or at banks, and students discussing numbers in the context of travel. Although some of the technology and travel responses had the potential to be considered in terms of rate of change, using only these questions, it would have been difficult to write personalized algebraic story contexts expressing linear relationships.

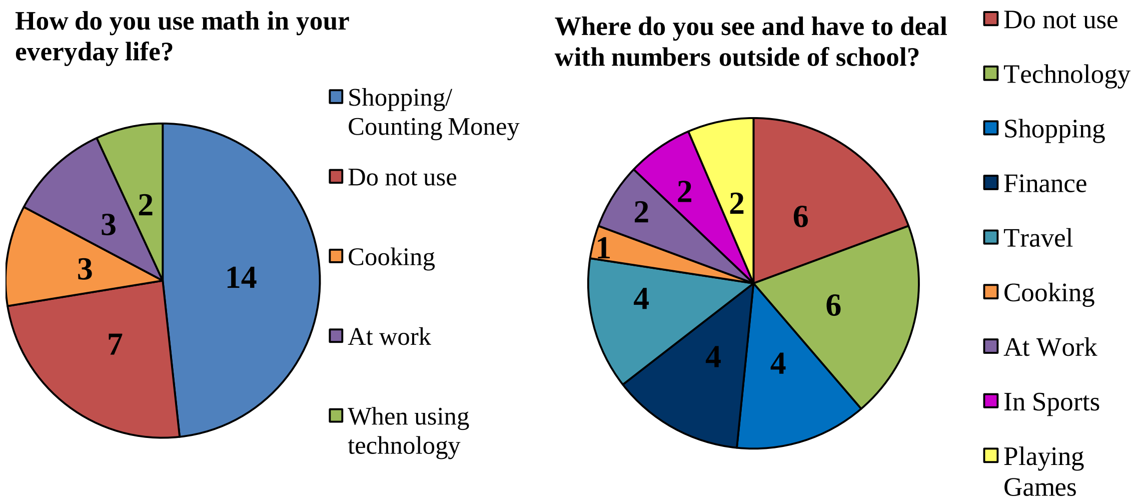


Figure 10. Distribution of student responses to two pre-interview questions (N=30 and N=32 student responses; one student's response could contain two different categories)

The teacher was also asked during her interview about how she used math in her everyday life; she responded "Yeah I mean definitely grading papers, figuring out the tip, figuring out my budget…" (Mrs. C, May 15, 2009). Similar to her students, the teacher mentions primarily arithmetic scenarios. Accordingly, when the teacher was asked if she used concepts from algebra in her everyday life, she responded "Not particularly. Not that I can really think of off-hand" (Mrs. C, May 15, 2009). The teacher was also asked how she thinks her students use math in their everyday lives, and she mentioned that they likely used math to budget their allowance and figure out their grades, but that they were not likely to use concepts from algebra outside of school beyond general problem solving.

Chazan (1999) describes his experiences as an algebra teacher using a traditional curriculum and then using a "functions-based" approach. One method he used to relate algebra to students' lives was to have students "identify the aspects of their experience which could be, at least theoretically, measured, counted, or computed from other quantities" (p. 127). This approach was used in the interviews presented here, and students were engaged in discussions like the one below, where a student describes how numbers are used in one of his favorite video games:

S: There's stuff like, this unit has 1000 health and does 100 damage per attack. And then the other units have they might have 10,000 health and they might to 20 damage per attack. If I have them attack each other, who will win? And other stuff... if you're on the high ground, you have, they only have a percent chance of hitting you. So if you're on the higher ground, and you're weaker than they are, you may still win the battle.

I: What's the percent chance of hitting you?

S: 60% . They all won't hit you, because they can't see you, and they're just firing at random.

Although the majority of responses from students were not of this quality, discussions with most students in the study resulted in at least two everyday scenarios that they may think about in terms of rate of change. Many of these scenarios ended up relating to students' use of technology; numbers and rate of change seemed to fit nicely into experiences relating to computers and video games.

F. *Methods of Analysis*

The 24 student interviews were transcribed in the NVivo Qualitative Analysis software, and put in blocks such that one block of the transcript was a student working one part of one question (i.e. a problem-solving block) or a student answering an interviewer question. Since most students worked five four-part problems as part of their interview, each transcription had approximately 20 problem-solving blocks. However, if a student came back to a problem part later in the interview after initially completing the problem, it was considered a new transcript block. Overall, the transcriptions contained 486 blocks where students were working problem parts, and 164 blocks where students were answering interviewer questions. Students' written work was integrated with the corresponding problem-solving block. One audio file was destroyed shortly after the interview, and another interview was cut off during the student's solving of their final problem when the recorder ran out of power. In both cases, student work and interview notes allowed the lost interview problems to be used in the analysis in a limited manner.

Each block of the transcription was coded with hierarchical coding categories, including what base problem the student was working (see Appendix B), what problem type the student was working (see Table 3), and what problem part the student was working (result unknown, write equation, start unknown, interpret parameters, write story, answering interviewer questions). A set of problem-solving coding categories (Table 4) were identified from the data using the constant comparative method (Glaser & Strauss, 1967), where the main researcher went through several iterations of coding every interview.

The final categories included whether the student arrived at an intended or unintended answer, what strategies were used and what mistakes were made, evidence that participation practices from everyday situations were explicitly used during problem solving, issues with verbal interpretation of stories, students' use of non-coordinative approaches, and the creation of a symbolic equation disconnected from how the student solved other parts of the problem. Student solutions were classified as being "intended" and "unintended" versus "correct" and "incorrect" because in the second pilot study there was evidence of sound, but unintended reasoning about story scenarios which led students to alternate solutions. See Appendix D for the specific criteria used to assign codes. Two individuals who were math education graduate students and mathematicians independently coded these categories in a sample of 7 of the 24 interviews, and obtained kappa values of 0.79 to 0.96, which is between substantial and almost-perfect agreement (Landis & Koch, 1977). Table 4 shows the kappa values for each coding category.

Kappa measures inter-rater agreement for categorical items, and is considered a conservative measure because it takes into account that raters may agree by chance; simple percent agreement does not acknowledge this possibility. A kappa value of 1 is perfect agreement and a kappa value of 0 implies that raters agreed exactly as much as would have been expected if they were randomly selecting coding categories. Kappa is calculated by taking the observed agreement and subtracting the probability of random agreement, and then dividing this difference by the complement of the probability of random agreement. Discrepancies in coding for the set of 7 interviews were resolved through discussion, with reliability data recorded before discussion. The remaining 17 of the interviews were re-coded one final time when the inter-rater analysis was complete.

Coding Category	**Possible Codes**	**Kappa**
Outcome of Problem Part	*Intended Answer* *Unintended Answer* *No response*	0.94
Result Unknown Strategies	*Use Arithmetic* *Use Symbolic Equation*	0.87
Start Unknown Strategies	*Trial and Error* *Unwind* *Solve Equation* *Proportional Reasoning* *Repeated Addition* *Other*	0.80
Result Unknown and Start Unknown Mistakes	*Arithmetic mistake* *Forgot slope* *Forgot intercept* *Mixed up slope and intercept* *Mixed up result unknown and start unknown* *Took into account movement* *Applied invalid proportional thinking* *Other*	0.81
Write Equation Mistakes	*Too general* *Too specific* *Inverted operation(s)* *No independent variable* *Mixed up slope and intercept* *Forgot intercept* *Other*	0.96
Use of Non-Coordinative Reasoning	*Present* *Not Present*	0.85
Issue with Verbal Interpretation	*Present* *Not Present*	0.79
Use of Participation Practices from Everyday Situations	*Productive Use* *Unproductive/Disruptive Use* *No explicit use*	0.84
Symbolic Equation Disconnected	*Present* *Not present*	0.80

Table 4. Problem-solving coding categories with kappa reliability values (2 raters)

Additional coding categories were developed to describe students' responses to a series of pre-determined questions that were posed in almost every set of interviews (Table 5). The response categories for each of the questions were decided using the constant comparative method. For the pre-determined question categories, both coders independently coded all instances of students answering these questions across the 24 interviews, and obtained kappa values of 0.89 or above. Discrepancies were resolved through discussion after reliability data was recorded.

Pre-Set Question Coding categories	
Coding Category and Possible Codes	**Kappa**
How do you use math in your everyday life, when you're outside of school? *(Shopping, cooking, at work, technology, don't use)*	1
Where do you see and have to deal with numbers when you're outside of school? *(Technology, shopping, finance/business, at work, nowhere/other)*	0.89
What do you think is the purpose of learning algebra in high school? *(finance-related, future job-related, future high school or college courses, nonspecific need for future/don't know)*	1
What do you think is the purpose of learning about story problems in algebra class? *(useful in everyday situations, useful in workplace, useful on standardized tests, creative thinking, don't know)*	1
Which of the problems that you worked did you find easiest? Why? *(mathematical structure, relatable to my life, other)*	0.92

Table 5. Coding categories for pre-set questions posed during interviews, with kappa reliability values (2 raters)

VII. Chapter Seven: Results and Discussion

A. *Comparative difficulty of problem types*

The first outcome examined here is the comparative difficulty of the different problem types presented to students. Students obtained the intended answer for approximately half of the questions posed to them, with abstract and normal problems being most difficult (see Figure 11). In terms of problem parts, result unknowns were the easiest, followed by start unknowns and writing equations (see Figure 12). These results are similar to findings from larger quantitative studies (e.g. Koedinger & Nathan, 2004) which have shown that verbally-presented problems are easier than abstract or symbolic problems, and that result unknowns are easier than start unknowns. However, it is interesting to note here that normal story problems had close to the same success rate (45%) as abstract symbolic equations (46%). In terms of problem parts, interpreting the parameters (slope and intercept) in a given equation in the context of a story scenario, and writing stories based on equations were most difficult.

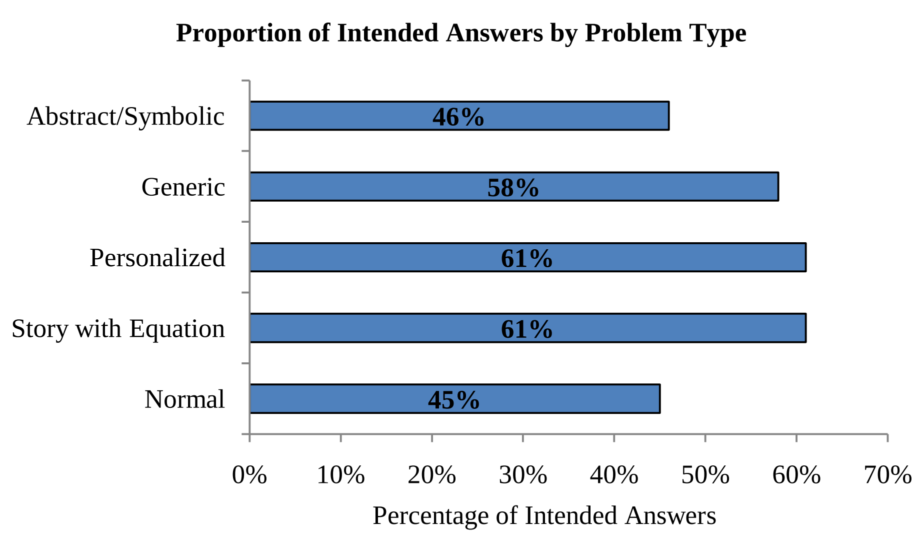


Figure 11. Percent of responses to each problem type that were intended (correct). Only includes result unknown and start unknowns problem parts - these were the only parts consistent across problem types (N = 33 abstract, 47 generic, 152 personalized, 61 story with equation, and 74 normal problem-solving blocks)

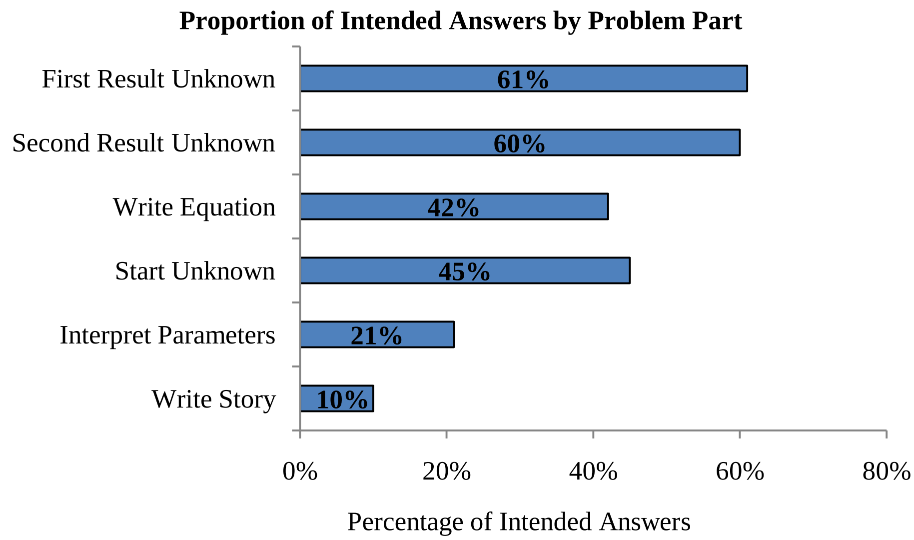


Figure 12. Percent of responses to each problem part that were intended answers (N= 123 first RU, 132 second RU, 92 write equation, 110 SU, 19 interpret parameters, and 10 write story problem-solving blocks)

This research study will not report data analysis aimed to systematically determine which problem types were more or less difficult than others in terms of correct answers. A quantitative analysis of this data has been conducted in other related work, and a quantitative randomized control experiment with a much larger sample size was also conducted for this purpose. The present study instead gives rich, qualitative findings on how different story scenarios interact with problem solving in mathematics, and the relative difficulty of problem types and problem parts is meant to set a broad context for this discussion.

As part of the teacher interview, the teacher's beliefs about the difficulty of the story problems being presented to students were investigated. The teacher was given two of the most difficult normal problem scenarios to solve (base problems 3 and 5 in Appendix B) and was asked what proportion of her students would be able to successfully solve each one. For the first problem (base problem 3), the teacher estimated that "maybe half, without any sort of pre-lesson or anything" (Mrs. C, May 15, 2009) would be successful. This base problem was given to 10 students as either a normal problem, a personalized problem, a generic problem, or an abstract problem. The actual success rate was 32% overall, and 22% for the normal problem type that the teacher was given. For the second problem (base problem 5), the teacher estimated a success rate of 75-80%, whereas the actual success rate for the 11 students who received this problem was 31% overall, and was 20% for the normal problem type. The teacher's difficulty estimating students' ability to solve difficult story problems without a warm-up or recent direct instruction suggests that many students' knowledge of linear functions is more tied to the immediate lesson and school context than the teacher may have realized.

B. Students' conceptions of story contexts as promoting transfer

One justification for using story problems discussed earlier is that contextualized problems may allow students to transfer the knowledge they learn in mathematics classrooms to everyday situations and to the workplace. Interestingly, this was the justification most often cited by students when they were asked why they were given story problems in algebra class (see Figure 13, left).

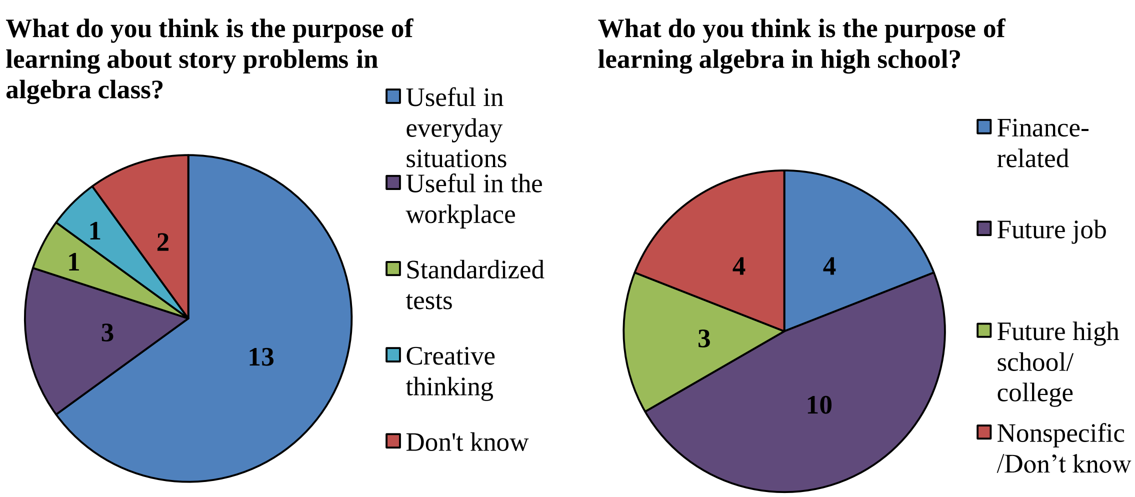


Figure 13. Students responses to two interview questions (N=20 and N=21 student responses; not all students were asked)

The majority (13/20 responses) of students responded that learning how to solve story problems would be useful to them in their everyday lives. Many of their justifications were financially-based, and mainly arithmetic in nature, such as the following:

> Well first you need to know if you have a bank account and you know, if they're messing up your money and you don't know, and you're just going along with it, you're not going to know. You're going to be short the money, and later in life you're going to be like 'Where's all my money?' And you just need to know it because it's... part of life.

Other students made connections to specific problem scenarios that they had solved during the interview "Because you can use it in real life, like the cell phone example you can use that in real life, because it could actually charge you that in life, yeah. So that's why they teach you that." These responses are similar to the teacher's response when she was asked why students are given story problems to solve in algebra class. The teacher referred to story scenarios as being useful for businesses like phone companies:

> I mean it puts it in a real context, so hopefully they'll be able to connect with it better, and see that they might not use this equation, but the cell phone company does. Or they have some equation that they use to figure out the bills. They don't sit there and figure out each person's bill individually, they can punch it in to the formula. (Mrs. C, May 15, 2009)

The next most common response from students (3/20 responses) when asked why they were given story problems in algebra class was that story problems relate to future careers:

> I: Why do you think you have to learn about problems like these in algebra? (indicates story problem)
>
> S: Because when you get a job, or you get older, they ask you about this right?
>
> I: So, like who would ask you about stuff like this?
>
> S: Like, well, if you get a job inside like as a teacher or anything, I'm sure they would. Maybe get a job like construction work, I don't think they would.

Students' responses when asked more generally why they had to learn about algebra were somewhat similar (see Figure 13, right). The most common answer (10/21 responses) was that algebra is useful in some jobs, specifically for architects, bankers, managers and business owners, cashiers, and mechanics. However students sometimes implied that these were not jobs that they personally were interested in. The next most common response (4/21 responses) was that algebra was useful for its financial applications, like paying taxes, paying bills, buying items, determining pay, and determining profit. Few responses (3/21 responses) were related to needing algebra for future high school or college courses. However, when the teacher was asked why students learn algebra, she had this focus, responding:

> I really think that it's to give them an idea of what's out there. Give them some options. That's why they have these core classes. They get a little taste of everything so that when they do go to college and they get specialized, they can decide, do I want to go into math? (Mrs. C, May 15, 2009)

Overall, the students in this study thought that the algebra they were learning in school would be useful to them for a variety of practical out-of-school and job-related purposes. The students appeared to subscribe to the “transfer to the real world” justification for learning algebra in context, although the research cited earlier has shown this idea of transfer to be problematic. The teacher seemed to recognize the transfer view during the interview when she suggested that students could see from a story problem how math was used by cell phone companies, however she more strongly accentuated the importance of learning algebra for future work in college, and the importance of story contexts to provide students with access or a “connection” to the problem. When asked specifically why she used story problems in her teaching, Mrs. C responded “To try and get that connection. To try and make them use that brain and see that this word problem is the same as an equation problem, you're doing the same thing in it” (Mrs. C, May 15, 2009).

It is interesting to note the differences between Mrs. C’s main justification for using story problems and learning algebra, and the responses given by the students. One student explicitly mentioned Mrs. C’s influence on his thinking, “Because we may use it for paying bills and like financial stuff... yeah she always says we need it, we always need math." Most of the algebra teachers at the school accentuated the real-life applicability of algebra to motivate students, and the students' textbook was filled with colorful graphics and application problems intended to show that algebra is in the world around them. However, when asked, Mrs. C accentuated the role of story problems in providing access, and the importance of algebra for future courses.

C. The accessibility of story contexts – verbal and situation facilitation

A second justification for using story problems is that contexts can provide students with access to important mathematical learning, through verbal or situation facilitation (Koedinger & Nathan, 2004). The following discussion describes how verbal and situational contextualization interacted with problem solving during the interviews.

The accessibility of different problem types

The interaction of context with problem accessibility was first examined by looking at "no response" errors across problem types. If context does provide students with increased access to problems, there may be fewer "no response" errors on story problems when compared to abstract problems, and fewer "no response" errors on personalized problems when compared to other story problems. The results are consistent with these hypotheses (see Figure 14), although it is interesting to note the lack of "no response" errors for the generic problems. This may imply that verbal difficulty of the problem influences whether students will respond.

Problem accessibility was also investigated by asking students at the end of the interview which problems they found easiest (Figure 15, left) and hardest. The vast majority of students (18/22 or 82%) responded that a personalized problem was easiest. Six of these students named a personalized problem where they had obtained an unintended answer for multiple problem parts. However, the most common justification given for a problem being easy (12/22 responses) related to the problem's structure and the operations needed to solve it (Figure 15, right). One representative response from this category was, “Because… maybe it was an easy form of an equation to use. It was just talking of money and the amount of messages you use so it’s just multiplying and that was easier.” The next most common justification for a problem being easy (6/22 responses) was that the problem related to the students’ lives and something they actually did. A student given a problem relating to his job at the flea market named the problem as easiest, responding, “Well, that made me remember what I worked on, so I did what I usually do at my work.”

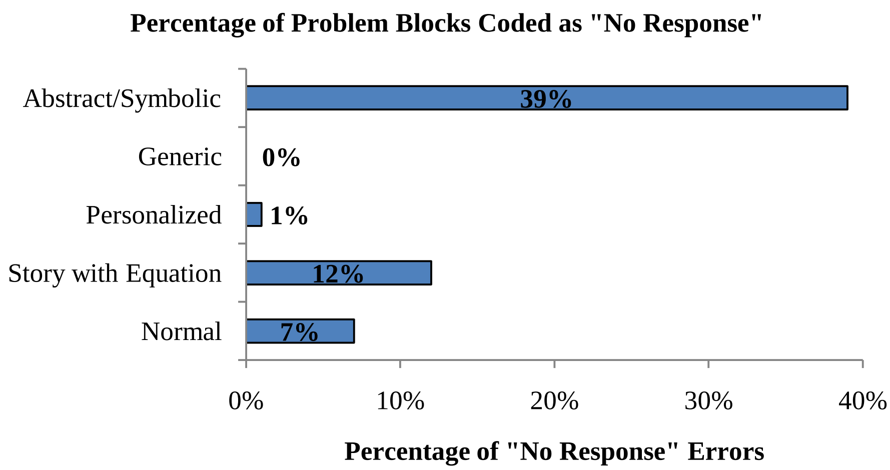
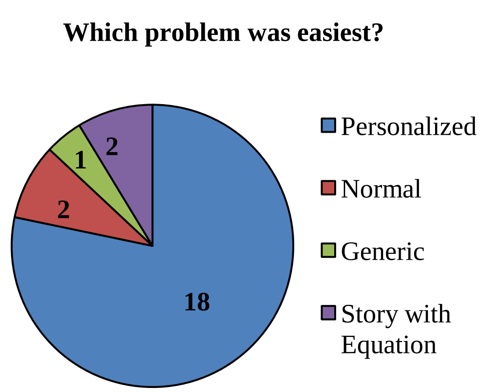


Figure 14. Percentage of problem blocks where students gave no response. Only includes result unknown and start unknowns problem parts - these were the only parts consistent across problem types (N = 33 abstract, 47 generic, 152 personalized, 61 story with equation, and 74 normal problem-solving blocks)



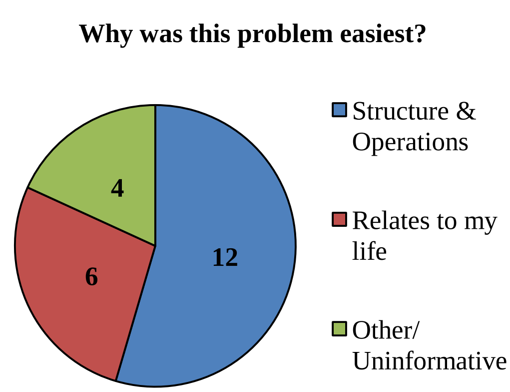


Figure 15. Students’ responses to two related interview questions (N=23 and N=22 student responses; one student named two problems as being “easiest”)

Abstract problems were most often rated as hardest (mentioned 67% of the time), followed by generic problems (mentioned 38% of the time), and story problems with equations (mentioned 33% of the time). Students sometimes named two of the problems in their set as being “most “difficult. Story problems with equations and generic problems were consistently named as being hardest, despite the fact that they had similar success rates to personalized problems (around 60% success), suggesting that problem framing may have motivational implications. One student directly addressed the interaction of motivation and problem framing when he named a generic problem as hardest:

> It didn’t give much information, and it's not interesting. Like, when the problem is interesting you want to figure it out because you’re curious to find out, it seems like it’s something that you want to know, you’re not just doing because you’re asked to do a question. A problem that you do because you truly want to know the answer to it, because you want to know that, you want to solve it, so it’s kind of easy… It’s not just for a grade or whatever it’s for.

However, once again, problems were most often described as being difficult based on their structure and the operations required when solving them.

The teacher also made some comments related to student access, motivation, and problem framing during her interview, suggesting that she does not believe that problem context is of primary concern to students:

> I: Okay, from your experience do students connect with the story problems that you use in your teaching?
>
> T: I think somewhat. I mean, I don't see that it's extremely important to them... they... you know they look at the word problems the same as they look at the bare naked problems. It's another math question, they're never too excited about it. But I do think that it does connect with them a little bit better.
>
> T: (later) So I think the equation problem would probably be harder - most people they skip those. Of course they skip the word problems too, but...
>
> I: Your students skip the ones that are the straight (equations)...?

T: If they aren't multiple choice, yeah they'll skip those. Granted they'll skip the word problems too. If the word problems are too long they'll skim them, you know they'll skim them and see, is it too hard? If it is they'll move on. (Mrs. C, May 15, 2009)

In summary, there were low “no response” rates on personalized problems and students considered these problems to be easiest, even though they often justified this rating structurally. Abstract problems had high “no response” rates, and were often named as hardest, along with story with equation and generic problems. Students’ conceptions of problem difficulty seem to be tied to the verbal framing of the stories.

Verbal interpretation of stories

Koedinger and Nathan (2004) acknowledge with the verbal facilitation hypothesis that algebra students “have by now mostly mastered the English comprehension knowledge needed for matched verbally stated problems” (p. 138). However, the participants in this study were consistently having a difficult time with the verbal component of the story problems. Instances were coded in the interviews where students expressed uncertainty about specific semantics of the story, where they stated an unintended inference based on the verbal language of the story, and where the student did not understand a word in the story. Table 6 gives two examples of issues that students had with verbal interpretation. In the first case, the student’s difficulty understanding what the word “initial” meant caused him to ignore the slope term and use the intercept as the slope. In the second example, the student concluded based on the wording of the story that the intercept term was not significant in the context.

Issues with verbal interpretation were found in 79 of 486 applicable blocks (16% of the time). Out of the 24 students interviewed, 22 verbalized at least one issue with story interpretation, and up to 7 issues occurred during a single interview. The prevalence of verbal issues may be in part a result of the student population selected. The school had a high Hispanic population and for 5 students in the study, Spanish was the language spoken by the guardian that gave consent for participation in this study. However, the average number of issues a student from a Spanish-speaking home had with verbal interpretation was similar to the overall average, with 2 of the 5 students having one or no issues, and the other 3 students having 4, 6, and 7 issues each. If a student had an issue with verbal interpretation in a problem-solving block, their chance of getting the intended answer to that problem part was 29%, compared to a 51% overall success rate.

Description of Issue	Problem Being Worked	Interview Transcript
Interpretation of mathematical vocabulary	Some rental cars have mobile phones installed. In one car, the cost of making a call from the mobile telephone is $1.25 per minute with an initial fee of $2.50. If a call cost a total of twenty dollars, how many minutes did the call last?	I: Can you... do you know what this word here means? S: No. I: OK, "initial"? OK. And what about the 1.25? How come you didn't use that one? S: The 1.25 is the…per minute, how much it costs per minute. (long pause) Per minute, but I think that "initial" is plus tax, or the whole thing together.
Start position confusion	A huge mirror for a telescope is being moved by a truck with 13 axles and 50 tires, from Erie, Pennsylvania to Raleigh, North Carolina. The truck averages 15 miles per hour and has already traveled 60 miles. In three more hours, how many miles will the truck have traveled?	I: Can you tell me, what's up with that 60 there? S: You've already traveled that much... so I would have to add on this? I: Do you think that's something that needs to be taken into account in these, or do you think it's not? S: When I read it, because it's just saying that he went 15 miles per hour, but he's already just traveled 60. So it's just like checking in, kind of.

Table 6. Examples of students' issues with verbal interpretation from interviews

Students had issues verbally interpreting story problems even when the story problem had been personalized to an experience they described during the entrance interview. Issues with verbal interpretation occurred 16% of the time with personalized problems, compared to 10% of the time with generic problems, 19% of the time with story with equation problems, and 23% of the time with normal problems. These results suggest that for many students, story problems do add verbal comprehension demands, even at the algebra level, and that these verbal issues have an impact in terms of problem-solving success. The prevalence of verbal interpretation issues seems to undermine the idea that story problems promote connections between in school and out-of-school participation practices, and thus that verbal framings always provide students with access to mathematical ideas.

Using participation practices from everyday situations when solving story problems

The idea of "situation facilitation," or that students can use their knowledge of the "real world" to directly help them solve story problems, has been central to previous studies of algebra problem solving (e.g. Nathan et al., 1992; Koedinger & Nathan, 2004). Here, situation facilitation was framed as students' ability to productively apply participation practices they use informally in everyday situations to the solving of story problems in the mathematics classroom. In order to critically examine this idea of situation facilitation, instances were coded in the interviews where students explicitly generated inferences from their experiences that were not given in the problem text.

Instances were only coded for result unknown, start unknown, and write equation problem-solving blocks. "Interpret parameter" and "write story" problem parts had been explicitly designed to require students to use their situational knowledge for a response to be possible, thus these blocks were not included to avoid confounding the prevalence of this behavior. These problem parts and their relationship to students' use of participation practices from everyday situations are discussed later in a separate section. The discussion here focuses on instances where students spontaneously generated inferences based on situational knowledge while solving problems more typical of school algebra.

Overall only 20 instances of students explicitly using participation practices from everyday situations were found out of 457 applicable blocks (4% of time); 10 of the 24 students explicitly used situational knowledge while solving a problem between 1 and 5 times each per interview. Instances were coded as being productive or unproductive/ disruptive with respect to the "intended" solution path of the problem, i.e. a solution path leading towards an answer that would be valued in a school mathematics context. Of the 20 instances found in the data, 9 were coded as productive and 11 were coded as unproductive or disruptive. Table 7 shows several examples from the data.

In the first example in Table 7, the student uses the fact that he got a decimal answer for "number of objects" as a signal that he made a mistake. Students' recognition that their answer was unreasonable in the context of the story was the primary way in which situational knowledge was used productively. In the second example, the student attempted to reason with situational knowledge, but it was not helpful; she was trying to see if there was a relationship between a problem she had recently solved in her everyday life about figuring out how many hours of dance she needed to do each day, and the problem about MySpace she was being asked to solve. The two scenarios were incompatible in their structure – her real-life problem about dance hours involved division and taking into account a 5-day week structure, while the problem on MySpace involved a combination of multiplication and addition and was framed simply in terms of days.

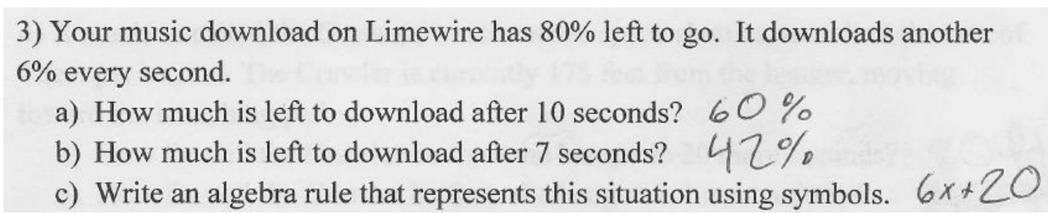
Description	Problem Being Worked	Interview Transcript
Productive	You have 80 objects, and lose 6 every day. After how many days will you have 8 objects?	I: Why wouldn’t you want to get a decimal for the answer to this one? S: Because you can’t lose half an object. Because if you have a toy and you lost half of it, it doesn’t make any sense.
Unproductive	You have 175 friends on MySpace. You get 4 more friends every day. How many total friends will you have in 20 more days?	S: I could do 175 times 20, from...OK…see this is how I was thinking, because I have this dance camp, and for my dance class over the summer I’m going to have to have 20 hours, so I was saying, OK, so, and we have to do it for that 20 hours, so I was thinking, OK, I can do 2 hours every day for 2 weeks. And that comes out to 20 hours. So I was thinking about how I could do that for this one. So, I’m just like trying, to do it so, I’m just trying to figure out like, how I could that, for how I did that one. So, 2 times 5 is 10 a week, and then, just trying to figure it out like that…
Disruptive	The number of students getting A or B in algebra class is given by the equation y = .25x where x is the total number of students taking algebra. If 40 students earned an A or a B in Algebra last year, how many total students were enrolled?	S: (long pause) 80 students were enrolled. I: So how did you get 80 for that one? S: Just times the 40 students times 2, because there’s always a half that doesn't get the full stuff done, pretty much there’s so many students and then, it divides how many students get an A or a B, and the other students don’t get an A or B. So I guess it divides how many A’s or B’s I have.

Table 7. Examples of students explicitly using participation practices from everyday situations during interviews

In the third example, the situational knowledge the student applied was disruptive to problem solving, and led to an unintended answer. Rather than using the given equation, the student reasoned based on his experience with how grading normally works in algebra classes to determine that half of all students usually get Cs and Ds because they “don’t get the full stuff done.” The rate of students explicitly using participation practices from everyday situations was similar for personalized problems (3%), normal problems (2%), and generic problems (5%). Story problems with equations had a higher rate of students using everyday knowledge (10%), likely due to the fact that these were more impoverished stories that depended on the symbolic equation to express the relevant relationships between quantities.

Out of the 20 blocks coded as students explicitly using participation practices from everyday situations, 11 of these blocks were also coded as containing non-coordinative reasoning (i.e. reasoning demonstrating a disconnect between the situation described in the problem text, and the formal mathematical operations used the solve the problem). The relatedness of non-coordinative reasoning to students using situational knowledge seemed to happen for two reasons. First, students would choose to employ their everyday knowledge of the described situation *instead* of reasoning about the situation and relationships presented in the text, as in the third row of Table 7. Second, students would take a purely calculational approach, plugging in the given numbers in semi-random orders, and then use their situational knowledge to determine if their final answer was reasonable; this was the case in the first row of Table 7, which was considered a productive use of situational knowledge.

Another possible instance of situational knowledge preventing a student from getting the intended answer to a story problem is shown in the work of the student in Figure 16. While the answers to parts a) and b) in Figure 16 seem to imply that the student was not really reading the question, the answer to part c) is surprising. This student may have mentally transformed this problem into one that made more sense based on his experiences downloading music and movies; rather than the download box showing how much it has left to go, he seemed more accustomed to the idea of the download box showing how much had been downloaded so far. This is consistent with much of the arithmetic research showing that students' interest may cause them to think about the problem in different ways than intended, but that these ways can have validity. This study suggests that these types of conflicts occur not only for simple arithmetic problems typically encountered in everyday life, but also for more complex algebraic scenarios where students must reason in terms of a rate of change and intercept values. It was because there were so many cases of sound but unintended reasoning during the interviews that student responses were categorized as intended and unintended answers, rather than correct and incorrect answers.


3) Your music download on Limewire has 80% left to go. It downloads another 6% every second.

a) How much is left to download after 10 seconds? 60 %

b) How much is left to download after 7 seconds? 42%

c) Write an algebra rule that represents this situation using symbols. 6x+20

Figure 16. Student work showing alternate way of thinking about a story scenario

There were also instances in the interviews where students' use of participation practices from everyday situations would have helped students realize that their answer was unreasonable, but students chose not to take these considerations into account. For example, from the discussion of the second pilot study, Carl's work in Figure 8 shows that he answered that he could watch six 30-minute episodes of his favorite TV show in 6 hours - it is unlikely this answer would have made sense to him with more careful consideration of the problem scenario and the quantity being asked for. Another student calculated that it would cost $115.00 to send one hundred $0.23 text messages, while a third student said that an object moving at 1500 mph would move 16 miles in half an hour. A different student answered that he had downloaded "24.8 songs" for the given amount of money he had spent, while a final student answered that a $40 item on sale for 25% off would be $10.

Overall, the data demonstrate that while students can use participation practices from everyday situations to provide access to problems and error-catching benefits, explicit use is rare. When situational knowledge is used productively, it is often in the context of a non-coordinative approach to solving the problem. Situational knowledge can also disrupt students from the "intended path" of the problem, even in cases like the story problem shown in Figure 16 where the problem had been designed to match the student's out-of-school experiences based on an entrance interview.

Coding instances where students explicitly call attention to a connection they are making to everyday participation practices is a very conservative approach to evaluating incidence of situational knowledge being used. This coding was largely intended to provide insight into the ways in which students can use everyday participation practices, and the way in which these practices interact with the prevailing school mathematics system. Another way to examine how often students use participation practices from everyday situations when solving story problems is to look at patterns of strategy use across problem types, with a specific focus on strategies that rely on informal, situation-based reasoning. The next section discusses these strategies.

D. Informal strategies and issues of problem access

Strategy types for result and start unknowns

The strategies students used to solve result unknowns and start unknowns replicated findings of other studies of algebra problem solving. For result unknown problems, students used simple arithmetic approaches, such as multiplication, repeated addition, and proportional reasoning. Students also used symbolic equation approaches where they first translated the story into an equation (if the equation was not given), and then plugged the given x into the equation. Like in other studies, students rarely (6% of time) used a symbolic equation to solve result unknowns if the equation had not been specifically provided for them.

For start unknown problems, students sometimes used *trial and error* approaches where they plugged in different values of x to the equation or story, and tried to get the given value of y. Closely related to trial and error approaches were repeated addition and proportional reasoning approaches. In repeated addition approaches, students added the slope continuously, trying to reach the y value given in the start unknown, and then counted their repetitions to determine x. In proportional reasoning approaches, students "scaled up" previously solved result unknowns, trying to reach the given y value, and then looked at how much they had scaled to determine the corresponding x-value.

Trial and error, repeated addition, and proportional reasoning strategies are hereafter referred to as *informal strategies* to solve a start unknown. These approaches are closely related because they involve going forward in a functional relationship, and are tied to the precise action of the story or equation. Figure 17 shows a student's trial and error approach to solve a start unknown from the story scenario "Some rental cars have mobile phones installed. In one car, the cost of making a call from a mobile telephone is given by y=1.25x+2.50, where x is the number of minutes used. If a call cost a total of twenty dollars, how many minutes did the call last?" The student decides to use 3.75 as the slope term, and then systematically tries x values of 8, 7, 6, and 5 trying to find a number she can multiply by 3.75 to get 20.

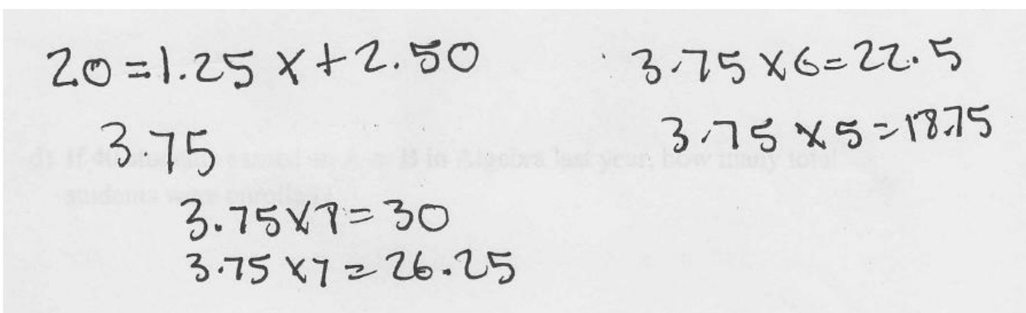


Figure 17. Example of a trial and error strategy to solve a start-unknown

Students also used *unwind* approaches to solve start unknowns, where they began with the given y-value, and then reversed the slope and intercept arithmetically. This is distinct from the previously mentioned informal strategies because there is some notion of reversing operations, and being able to systematically go backwards in a functional relationship. Here the unwind approach is considered to be a *transition strategy* for solving a start unknown, because reversing operations is tied to equation-solving strategies where students operate on both sides of an equation to isolate x. However, unlike with equation solving, there is no notion of balancing two sides of an equation.

Figure 18 shows a student using an unwind strategy to solve the problem, “You have 175 songs downloaded onto your iPod from Limewire and iTunes. You download 4 more every week. If you have 275 songs, how many weeks have passed?” The student successfully reverses both operations, but makes an arithmetic error.

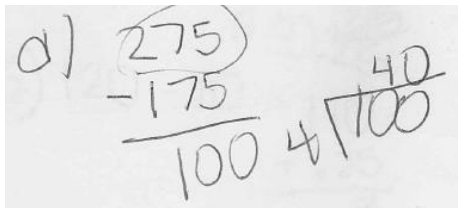


Figure 18. Example of unwind strategy to solve a start-unknown

The teacher correctly predicted during her interview that verbally-presented problems would be easier than symbolic problems. Her justification related to the “logical” or informal approaches students could use to solve verbal problems:

> I think the word problem would probably be easier, because they don't necessarily need the equation to do it, and they can just logically think about it to figure it out, if it's a word problem. But with the equation problem, they actually have to understand the variable. (Mrs. C, May 15, 2009)

Students occasionally used *equation-solving* approaches to solve start unknowns, performing operations on both sides of a symbolic equation to isolate the x variable. Students only used an equation-solving strategy between 6% and 22% of the time depending on problem type, despite the fact that for each problem they were either provided an equation or asked to write one in the problem part immediately before the start unknown. Solving the equation is considered a *formal strategy* to solve a start unknown; an example of this approach is shown in Figure 19.

Start unknown strategies by problem type

There were differences across problem types in the strategies students chose to use. Figure 20 shows the proportion of strategies used to solve a start unknown that were informal (trial and error, repeated addition, proportional), transition (unwind), and formal (equation solving) by problem type. Also included are how many "no response" errors there were and strategies that were classified as "other." The pie charts are organized such that moving from left to right, the concreteness and relevance of the problem context increases; abstract problems are least relevant and concrete, followed by story problems with equations, followed by normal story problems, and then personalized problems are the most relevant and concrete.

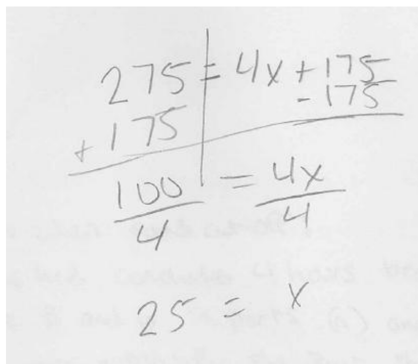


Figure 19. Example of an equation-solving strategy to solve a start unknown

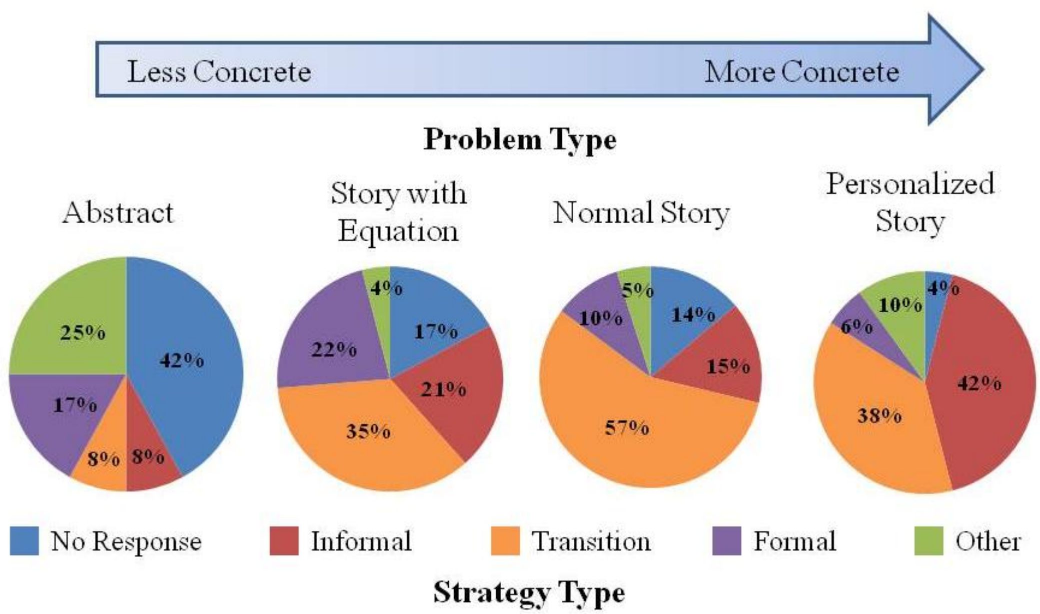


Figure 20. Prevalence of students' strategies for solving start unknowns, by problem type (N=12 abstract, 23 story with equation, 21 normal story, 50 personalized strategy codes)

As can be seen in Figure 20, moving from more abstract problem types to more concrete and familiar problem types, “no response” errors (blue) show a large decrease. Students’ use of informal strategies to solve start unknowns (red), such as the trial and error approach, becomes more prevalent as problem concreteness increases. Students use of the transition strategy unwind (orange) increases up to the normal story problems, then for personalized problems this strategy is overtaken by the informal strategies. Use of the formal strategy of equation solving (purple) is most prevalent in abstract problems and story with equation problems, and its incidence decreases moving to normal and then personalized story problems. The “other” strategy (green) included strategies that did not fit into any of the preceding categories; an example of an “other” strategy was mistaking a start unknown for a result unknown and solving it arithmetically.

These results showed an interesting trend in student strategies, however because of the small sample size, it was important to check if the pattern also held for individual students. The trend was investigated across individual students by looking at whether students changed between informal, transition, and formal strategies systematically by problem type. Of the 19 students who received either abstract or story with equation problems (pilot students did not receive these problem types), 12 used more formal strategies overall on these more abstract problem types, when compared to the strategies they used on personalized problems and normal problems. Only 2 showed the opposite trend, while 5 either used similar strategy levels across problem types or had too many “other” or “no response” strategies to make a determination. Overall, these results support the idea that individual students are changing their strategy use patterns based on the concreteness of the problem’s cover story.

These trends in student strategies may have implications for teaching equation solving in algebra courses using start unknown problems. Problems personalized to students’ experiences could be used to introduce a topic; they provide access for the widest range of students, and elicit informal, invented strategies. As students progress, they could move towards solving normal story problems, and then connect the informal and transition strategies they use on these problems to formal equation-solving approaches used in story with equation and abstract problem types. These ideas are consistent with other research in mathematics education showing the benefit of a “concreteness fading” approach to instruction (Goldstone & Son, 2005) as well as studies showing support for verbal precedence models (Nathan & Koedinger, 2004).

These results suggest that students choose different strategies based on problem framing, with more relevant and concrete problems eliciting greater use of informal, situation-based reasoning, and more willingness from students to attempt a solution. This implies that contextualization generally, and personalization specifically, may provide students with access to problems by allowing them to use their participation practices from everyday situations to informally reason through the operations needed to solve problems in an algebra classroom. However, these informal strategies seem somewhat at odds with a highly-valued practice in the school mathematics system – learning to manipulate symbolic equations. The next section presents a discussion of this tension.

Contrasting informal and formal strategies in story problems

Explicitly connecting informal, situation-based strategies like unwind and trial and error with the formal strategy of equation solving could promote learning equation solving with understanding, and the process of equation solving retaining meaning with respect to the story scenario. For example, the student whose work is shown in Figure 19 stood out from most of the other students in the study because she had apparently mastered equation solving. The problem she was solving read: “The distance a machine called the Crawler has traveled from its hangar is given by the equation y= 4x+175, where x is the number of seconds the machine has been moving. In how many more seconds will the Crawler reach the launching pad, which is a total of 275 feet from the hangar?”

Although she was able to obtain the intended answer to the start unknown using the equation, when asked immediately afterwards what the “4” could represent in the situation and what the “175” could represent, her response was that the Crawler could have started at 4 feet, and 175 could be the number of seconds it took the Crawler to move 4 feet. This response shows that while she obtained the “correct” answer and used perhaps the most valued strategy in the context of high school algebra (Nathan & Koedinger, 2000a), she lacked a fundamental understanding of central algebraic concepts like rate of change and intercept, and how these concepts relate to both equation solving and applied scenarios like travel. Current standards in math education have recognized that “In general, if students engage extensively in symbolic manipulation before they develop a solid conceptual foundation for their work they will be unable to do more than mechanical manipulations” (NCTM, 2000, p. 39).

Compare this to the reasoning of the student whose verbalizations are given below, who used an unwind strategy to solve the problem, “You have a Verizon cell phone and you have a gift card from Verizon with $7.87 left on it that you plan to use on this month’s bill. This month, Verizon is going to charge you $0.23 for each text message you send. At the end of the month, you pay $38.13 of your own money to Verizon. How many text messages did you send?”

> S: So there was 38.13, you’re gonna add the 7 from the card, plus 7.87… 38.13 plus 7.87 equals to $46, it equals to 46, and now you’re going to divide those, 46 divided by 0.23, which would equal to 200 text messages.
>
> I: Can you tell me why you added the 7.87 there?
>
> S: Yeah, because it’s telling you at the end of the month you pay 38 of your own money. So that’s not including the money that you already used from the card. So, and then it’s asking you how many text messages you send, so you need to get the total amount of money that you used to see how many text messages that you sent.
>
> I: So why did you divide the 46 by the .23?
>
> S: Because 0.23 is what they charge for each message. So you divide the total amount by the charge per message to give you the number of message you sent.

This student demonstrates a clear, conceptual understanding of the story scenario and how concepts of rate of change and intercept operate in this context, as well as why and how these operations are reversed when solving a start unknown. There is much potential with such reasoning to connect what students informally understand about story scenarios to more formal algebraic manipulations, especially in the case of the unwind strategy which is related to equation solving.

Some researchers dismiss students' informal arithmetic-based strategies; one study of algebraic reasoning concludes with "Under the guise of teaching algebra, some teachers promote non-algebraic methods because they believe they are easier for students. By doing so they fail to provide opportunities for students to learn more powerful mathematical methods" (Stacey & MacGregor, 1999, p. 164). However, an alternative view is that students' informal, situation-based reasoning is knowledge that can be built upon to provide access to algebraic ideas, similar to research at the elementary level on Cognitively Guided Instruction (Carpenter & Moser, 1984; Carpenter et al., 1999). In this series of interviews where exiting Algebra I students were only able to successfully solve start unknowns in simple linear equations 45% of the time, being able to solve a start unknown by any method is an important mathematical strength to be capitalized on.

The fact that the algebra students in these interviews were largely not comfortable using equation-solving techniques demonstrates that the way equation solving is being taught may not be connected with what students know and understand. In the classroom, most students in the study had become adept at "pretending" they knew how to solve an equation – if the teacher had recently demonstrated an example where a linear function was solved for x, students would be able to mimic her work, and perhaps with some additional assistance from the teacher were able to give the impression that they knew how to solve equations. However, in the interview setting, without a recently demonstrated procedure to copy, 19 of the 24 students used informal methods, even though the teacher had never demonstrated any of these strategies. More research needs to address how students' informal strategies and different problem framings can be used to support learning algebra concepts. Progressively less concrete problem representations may have the potential to scaffold students into using symbolic equations, and connecting informal, situation-based reasoning to equation solving may help students use these representations with understanding.

E. *Students' use of non-coordinative approaches*

Examples and prevalence of non-coordinative approaches

In arithmetic story problems, it is well documented that students sometimes use "direct translation" or "keyword" approaches to solve word problems (Hegarty et al., 1995; Jonassen, 2003). Nathan et al. (1992) allude to a similar "translation based approach," (p. 338) in their study of algebra problem solving, suggesting that poor problem solvers may bypass situational reasoning when solving a story problem, leading to omitted inferences and strategies or solutions that are inconsistent with the scenario.

There were a number of approaches to solving story problems during the interviews that are referred to here as *non-coordinative* – students seemed to be translating from the problem text to a problem model, without developing an elaborated intermediate understanding of the situation based on the problem text. Table 8 shows some examples of instances that were coded as non-coordinative approaches. Some students plugged in the numbers given in the story in seemingly random orders, using various operations, and trying to obtain an answer that "looked right" (see first example in Table 8). This was often done very swiftly in their calculator, however a shorter example is provided here, where the verbalizations are more clear and concise. Blocks were also coded as non-coordinative if the student was applying a well-known schema to a problem, when reading the problem would immediately reveal it did not fit that schema (see second example in Table 8).

Only result unknown and start unknown problem-solving blocks were coded with the non-coordinative approach code. Non-coordinative aspects of students' reasoning when writing equations were coded with a different category to be discussed later. Further, non-coordinative aspects of students' reasoning on interpret parameters and write story problem parts are discussed in more general terms in future sections. Around 39% of all unintended answers given to result and start unknowns during the interviews were coded in blocks containing non-coordinative reasoning. However, 20% of the time students engaging in non-coordinative reasoning in a block still ended up with the intended answer, compared to a 56% overall success rate for these problem parts.

Description of Approach	Problem Being Worked	Interview Transcript
Student plugs in numbers without intermediate situational reasoning	An object moves at 1500 mph. It has already moved 500 miles. How far will it have moved total 30 minutes from now?	S: Fifteen minutes… I: Can you tell me what you think this situation is about? Like, what's the story about? S: I have no idea. So, if it moves at one thousand five hundred miles per hour, that means…so I think I would just divide 500 and 30. I'm not sure. Alright, so 16. So maybe for that one, 16 miles? I: OK, can you tell me, explain why you divided 500 by 30? S: Because I'm not sure exactly what to do, but I think that if you divide the miles by the times then you'll get, so how much it's already moved by the time you'll get, the answer to it?
Student applies well-known schema to problem not fitting that schema	You're buying a new skateboard that is on sale for 25% off. If the skate board costs $44 normally, how much will you save?	S: You're buying a new skateboard that is on sale for $25 off, I mean 25% off. If the skateboard costs 42 normally, how much will you save? For this I need 44 times .25 to figure out, the percentage, like how much you take off, and subtract it by 44. That equals 11, so 44 minus 11.50 would be…the cost of the skateboard would be 32 dollars and 50 cents.

Table 8. Examples of non-coordinative approaches; students bypass forming a situation model from problem text

Of the 24 students in the study, 20 had at least one instance of non-coordinative reasoning during their interview. The pattern of non-coordinative reasoning for each of these 20 students was examined to determine if there were certain "profiles" of students who use these approaches. Eight students used non-coordinative reasoning on only 1 problem, 6 used non-coordinative reasoning on 2 problems, and 6 used non-coordinative reasoning on 3 or 4 problems. This suggests that students are not simply blindly applying the approach to every problem they solve; rather specific aspects of the problem context or mathematical structure may be cueing use of non-coordinative approaches.

Contrary to expectations, non-coordinative approaches occurred regularly even when the problem had been personalized; 13 of the 20 students who used non-coordinative approaches used them on one or both personalized problems they received, with 3 of these students using these approaches *only* on their personalized problems.

However, overall incidence of non-coordinative approaches was similar for personalized and normal problems (15% and 12% respectively), but was considerably higher for abstract, generic, and story with equation problems (between 26-28%). This seems to imply that more relevant, verbal contexts can promote lower use of non-coordinative approaches.

The interview setting and the fact that students may have felt that they needed to provide an answer for each question could have contributed to the overall prevalence of non-coordinative reasoning, as opposed to having more “no response” errors. However, this type of reasoning has been discussed in studies reviewed earlier, and was observed in the classroom, so the next section presents a discussion of *why* students may use non-coordinative approaches, and how this interacts with contextualization.

Reasons why students may use non-coordinative reasoning

One explanation for using non-coordinative approaches may be that students are not motivated to form a situation model, either because they think they cannot or because they do not want to. This justification seems to correspond with Nathan et al.’s (1992) idea that forming a situation model adds to cognitive demand, or Sweller’s (1994) notion of *cognitive load*. However, a related explanation is that perhaps students would like to form a situation model, but the verbal semantics of what is happening in the story are too difficult for them, so they resort to non-coordinative approaches. There was evidence of both reasons in the interviews. The student in the first row of Table 8 did not seem to be particularly interested in attending to the story scenario; however there were other students who read the story scenario repeatedly, had long pauses where they were pondering the story, but in the end used non-coordinative approaches. Approximately 27% of all non-coordinative blocks were also coded as instances where the student had an issue with their verbal interpretation of the story. The episodes in these blocks suggest that non-coordinative reasoning could be something students resort to when the verbal semantics of the story context make the formation of a reasonable and understandable situation model difficult.

A third reason why students may use non-coordinative approaches has received quite a bit of attention in the elementary-level literature, but has been largely unexamined in studies of algebra story problems. Inoue (2005) differentiates the "mindless calculational approach" of direct translation from a "conformist approach" where students suspend sense-making as a result of critically but perhaps unconsciously evaluating their experiences with school mathematics. The student in the second row of Table 8 may have been applying a well-known schema for solving a "percent off" school mathematics problem, and as a result may have bypassed carefully reading and making sense of the situation. However, a clearer example comes from a student presented with the problem shown in Figure 21.

2) Due to a billing error last month, Amanda has received a $7.87 credit towards next month's cellular phone bill. She pays a flat $0.23 per minute with no additional monthly charge.

a) If 43 minutes are used, calculate the bill for the next month.
b) If 100 minutes are used, calculate the bill for the next month.
c) Write an algebra rule that represents this situation using symbols.
d) After finding that billing error last month, this month Amanda will make sure that her bill is correct. If her bill is for $38.13, how many minutes has she talked?

Figure 21. Example of problem that student believed to be multiple choice

Although the interviewer did not realize it initially, the student operated throughout the interview under the assumption that the problems he was being given were multiple choice. The interviewer was understandably confused by his insistence that the answer was "d" in the excerpt below.

> S: (a) says if 43 minutes are used, calculate the bill for the next month. And I was just putting in 43 times .23. I came up with 989 so that doesn't work. Or 9.89, so that doesn't quite work.
>
> I: Why doesn't that work?

S: Because I think it's trying to find, like, how many minutes she can be on the phone to add up to 7.87. And for (a) it says 43 minutes, and I came up with 9.89, so…

S: And (b) is wrong because it's even more minutes than 43 minutes.

S: I'm gonna go with (d) because it says if her bill was 38.13 dollars, and it's trying to find how many minutes she's talking and so you just divide it by .23, and I came up with 165.7 minutes. Cuz, the total bill $38.13 and that divided by .23, I just came up with 165.78, so I'm just going to go with (d).

This student's interview took place the week before the state standardized test for 9th grade mathematics, which is a multiple choice exam. This was one of the most mathematically competent students in the class; he was able to write the intended symbolic equation for this scenario when prompted, even though this was one of the more difficult problems in terms of mathematical structure. His school had been "Academically Unacceptable" in mathematics the previous year, and was facing significant sanctions if it did not improve student performance on standardized tests. Classroom observations showed that as a result, algebra students in the classes participating in this study were drilled on multiple choice standardized test style algebra problems starting the first week of school, with increasing intensity as the state test approached.

The fact that a mathematically competent high school student seriously believed that the problem in Figure 21 and the other problems he was given were multiple choice, and attempted to reason through these problems under that assumption, points to the importance of considering the larger system of school mathematics that students are participating in when they solve story problems. This example also seems to undermine the idea that the purpose of story problems is to help students gain an understanding of how to apply their learning in school mathematics to other systems of participation. The idea that mathematical problem solving is about choosing the best alternative from a list of possible answers that make little sense is not a problem-solving practice that would be valued or useful in any setting other than school mathematics.

These results suggest that non-coordinative reasoning was a significant part of problem solving in this traditional algebra classroom, and that simply personalizing a story problem to students' interests and experiences may not be enough to combat wide use of these approaches. Students' use of non-coordinative approaches undermines both justifications for putting mathematics in context; if students are not trying to make sense of the story for whatever reason, use of verbal and situational knowledge to support problem solving seems unlikely, as does transfer of school-learned problem-solving skills to other contexts. Non-coordinative reasoning is critical to any discussion about contextualization, because these approaches are an artifact that is unique to the school mathematics system. Their prevalence demonstrates the problematic epistemological statements about the knowing and doing of mathematics that are communicated in school, and causes concern for the degree of conceptual sense-making students engage in around central concepts in algebra as they apply to modeling the world.

F. The interaction of story contexts with symbolic representations

As the majority of research on story problems has been done using arithmetic scenarios, there has been less research on how story contexts interact with symbolic representations, using situated perspectives to understand the norms and purposes of representation. In all the problems that were given to students in the present study, a symbolic representation was provided or students were explicitly asked to generate a symbolic representation from a story context. The next section discusses the cases in which symbolic equations were provided.

Students' conceptions of symbolic equations provided as part of the problem

Two problem types were used in the interviews that contained symbolic representations as part of the problem text; the story problems with equations and the completely abstract problems (see Table 3). An interesting finding shown earlier in Figure 14 is that while "no response" errors occurred 40% of the time when students were presented with abstract problems, "no response" errors only occurred 11% of the time when students were given story with equation problems. Out of the ten students in the study that were given both story with equation and abstract problem types, three of these students refused to work their abstract problem, at least initially, but were willing to work their story with equation problem. One of the three students, after solving his story with equation problem successfully, was willing to go back and try the abstract problem, perhaps seeing the parallel between these two problem types.

Another one of the three students was first presented with the problem “The price of installing wall-to-wall carpet in your house is given by y=12.95x, where x is the number of yards of carpet.” This student proceeded to successfully solve both result unknowns and the start unknown problem parts, and gave a reasonable interpretation that the slope parameter 12.95 could represent “how much it is by the yard.” However, two problems later the same student was presented with the problem “y=2x” and was asked to solve for y if x was equal to 3. The following conversation occurred:

> S: Oooh... if x equals 3, what is y? I don't like these problems! I don't know how to do these problems.
>
> I: Okay. Can you tell me what that means there (points to equation)? Or what you think it might mean?
>
> S: I don't know. y=2x... what do you mean, what does it mean?
>
> I: Just when you see that, what do you think about? (pause) Think that you just don't know?
>
> S: Mhmm.
>
> I: So you you think you can solve any of these, or no?
>
> S: Probabley not.

What is significant here is that there is no way to solve the story with equation problem correctly without dealing with the symbolic equation in the exact same way it must be dealt with in an abstract problem– there is no redundant information relevant to solving the problem that the story scenario adds, it is merely “decorative” in this case. The prevalence of “no response” errors when presenting students with completely abstract algebraic problem types has been documented in other studies (Koedinger &

Nathan, 2004), however the results here suggest this is not simply a result of students not being capable of dealing with symbols. There seems to be a psychological effect for imbedding an equation within a few sentences of verbal context that allows students to better access this abstract representation. Story problems with equations have not been part of many studies, and may be a useful bridge between students' informal understanding of verbal scenarios and their ability to interpret and use symbolic expressions. There were higher success rates (Figure 11), higher response rates (Figure 14), and a greater variety of strategies (Figure 20) for story with equation problems than abstract problems. This novel result lends support to the idea that concrete, verbal contexts in story problems can provide students with direct access to mathematical formalisms like symbolic equations.

One might assume that students' higher success with the story with equation problem type is because they were able to form a situational understanding that supported problem solving and the formation of a problem model. However, of the 19 students that were asked to interpret the parameters (slope and intercept) in the equation that accompanied a story with equation problem, only 4 gave responses that demonstrated a clear understanding of how the equation related to the given situation. Students struggled to understand the slope parameter as rate of change. For instance given the scenario "The total distance the explorers have traveled is given by the equation $y = 20x$, where x is the number of days they've been traveling" one student responded that the 20 represented distance, rather than interpreting 20 as a relationship between distance and time. Sometimes new quantities were invented for the parameters; another student when asked to interpret the parameters from the scenario "The distance a jet has flown in miles is given by the equation $Y=1500x + 500$ where x is the number of hours the jet has been flying" said that 1500 was how high the jet was.

Overall when asked to interpret parameters, students' responses did not support the notion that embedding the equation in a story scenario enabled students to form coherent situation models that supported problem solving. Students' reasoning was often non-coordinative in nature, in that the connection between the situational context (situation model) and the symbolic equation (problem model) was weak.

Students' success writing symbolic equations to correspond to story scenarios

For each personalized, normal, and generic story problem, after solving two result unknowns students were asked to write a general equation corresponding to the story scenario before solving the start unknown. Students wrote a total of 85 equations from story contexts as part of this study. Students had some difficulty using story scenarios to write equations; the success rate for this problem part was 42%. By problem type, the success rate for writing an equation to go with a normal problem was 35%, with a 43% success rate for personalized problems and a 53% success rate for generic problems.

Several students, like Matt in the second pilot study, were initially confused by the idea of writing a symbolic equation to go with a story scenario. The teacher acknowledged students' difficulties writing symbolic equations when she was asked which problem part students would find most difficult (part "c" mentioned below corresponds to writing the equation):

> I: Which part of the problem, a, b, c, or d do you think your students would find most difficult?
>
> T: It's a tough call between c and d. I mean c is just putting a variable in there... like in a and b, and d of course makes you work backwards. But they always have a really hard time putting variables in.
>
> I: Why do you think they have so much trouble putting the variables in?
>
> T: They don't like variables. They don't like having something that isn't set in stone. They don't like gray areas. It's either black or white... that's really how their minds work. It's either right or wrong. (Mrs. C, May 15, 2009)

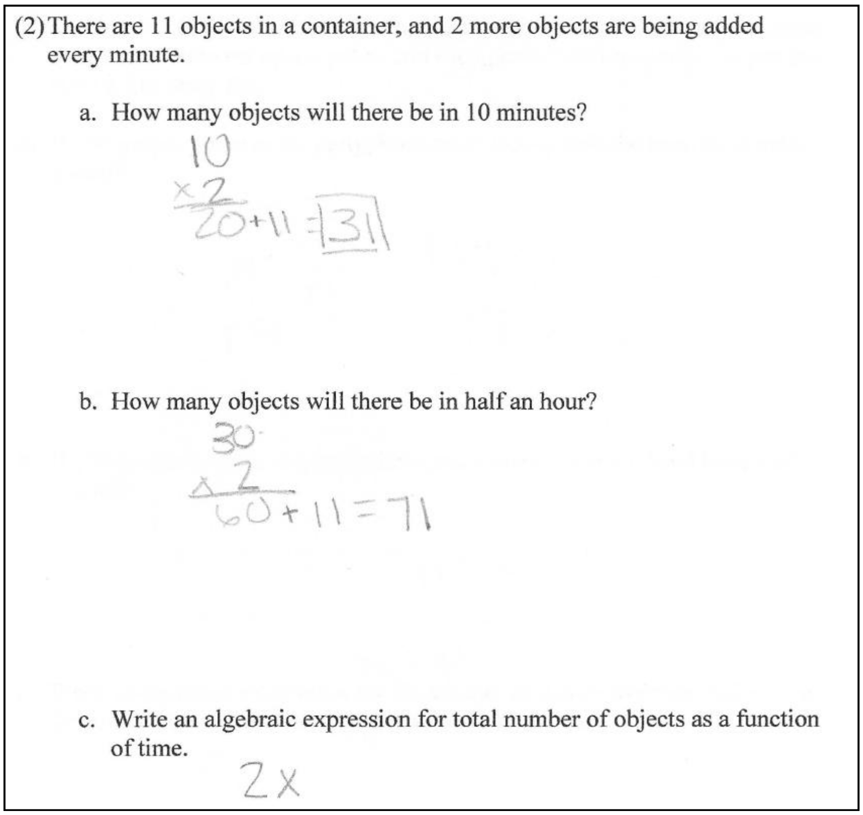
The most common mistake students made when writing an equation or expression was to leave out the intercept term (16 out of 62 mistakes). Interestingly, forgetting the intercept term was also the most common mistake students made when solving result and start unknowns in story contexts. Students seemed to have a strong implicit understanding of relationships between quantities that were directly proportional, but including an additive “start value” as part of that relationship caused students a great deal of difficulty. In many of the “real life” scenarios that algebra story problems are typically embedded in, it can be unclear why precisely the intercept term is significant. For instance, in a problem involving movement at a specific rate, the intercept term may be framed in terms of how far something has moved at a given, relatively arbitrary moment; this was the case with the normal problem in the second row of Table 6, and with some problems discussed in the second pilot. Normal problems had a high incidence of students not using the intercept term (27% of blocks), while personalized and generic problems had a lower, but still substantial, incidence of this behavior (13-14% of blocks). This suggests that more relevant contexts and contexts that are less demanding in terms of text interpretation may be more intuitive for students to think about in terms of rate of change and intercept.

Another common mistake (9 of 62 mistakes) was to write an equation that was too specific, i.e. where concrete numbers were used instead of variables. If students view a variable as simply being a “missing number” (Chazan, 1999) rather than a set, then it makes sense that the students would specify exactly what the missing numbers were, rather than represent them symbolically.

The disconnect between symbolic representations and situational reasoning

Several students used the intercept term to solve one or more of their result unknowns, but did not include the intercept term when writing the corresponding equation; an example is shown in the student work in Figure 22. Some students also made the opposite mistake – they would *not* use the intercept term when solving result unknowns, but when they wrote the equation, suddenly the intercept term would appear; an example is shown in the student work in Figure 23.

For many students, it seemed that writing the equation, or forming an explicit problem model in the way most valued in school algebra, was disconnected from the situation-based reasoning they used to actually solve result unknowns and start unknowns. Students were not tied to the belief that the problems they were being presented with had to make sense from problem part to problem part, suggesting a weak connection between situational reasoning and formal problem-solving procedures. Around 34% of all equations written were coded as being clearly disconnected from how students solved the other problem parts.


(2) There are 11 objects in a container, and 2 more objects are being added every minute.

a. How many objects will there be in 10 minutes?

$$\begin{array}{r} 10 \\ \times 2 \\ \hline 20 \end{array} + 11 = \boxed{31}$$

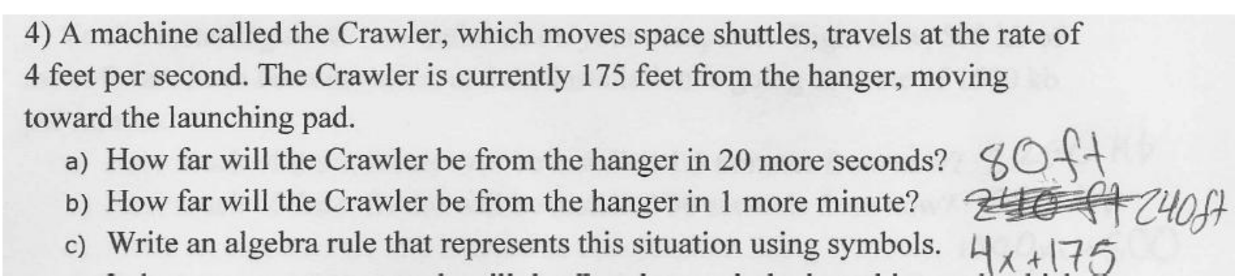
b. How many objects will there be in half an hour?

$$\begin{array}{r} 30 \\ \times 2 \\ \hline 60 \end{array} + 11 = 71$$

c. Write an algebraic expression for total number of objects as a function of time.

$2x$

Figure 22. Example of student work where intercept is taken into account in result unknowns, but not equation


4) A machine called the Crawler, which moves space shuttles, travels at the rate of 4 feet per second. The Crawler is currently 175 feet from the hanger, moving toward the launching pad.

a) How far will the Crawler be from the hanger in 20 more seconds? 80 ft

b) How far will the Crawler be from the hanger in 1 more minute? ~~240 ft~~ 240 ft

c) Write an algebra rule that represents this situation using symbols. $4x+175$

Figure 23. Example of student work where intercept is taken into account in equation, but not result unknowns

However, there were occasional cases where students were using the symbolic equation as a way to make sense of the story scenario. Two students, like the one who is quoted below, actually corrected mistaken result unknowns after writing the equation:

> I: Do you have any idea what you were thinking differently before that caused you to get different answers, or…?
>
> S: I think I was thinking differently because I hadn't thought about the equation, and that's what made it make the difference, because once I thought about the equation, all I really had to do was plug in the number and that's all.

Some students were also asked during their interviews why they thought it was important to write a symbolic equation to go along with a story problem. The responses were generally action-oriented (Breidenbach et al., 1992) – writing an equation was important because it reminded you of the string of operations you needed to perform to get from a specific given x value to the "answer." Below is an action-oriented response from a student who had solved a story scenario that had the equation $y=4x+175$:

> Because it'd help you, at first it has the starting point so you always know, cuz you could... you wrote it down, and then it says how much you add or subtract of each thing, because… you'd add four every minute, so it's in the equation, so then if two minutes have passed you just do 175 plus four, in parentheses, two, and another parentheses.

For many students in the study, equations were not an expression of a systematic relationship between two variables; they were a string of arithmetic operations. A similar response was, "So no matter what number you get, we have a way of solving it, when whatever number they give you, you solve it using… you can plug in any number and solve the equation." The second student shows some initial understanding of a variable as a set rather than simply a missing number (Chazan, 1999), but still seems to think of an equation as a way to get an answer by going forwards, and as generalized arithmetic.

One student alluded to the efficiency of symbolic representations saying that the equation helped "just to do the mathematics, sometimes, instead of having to go through the whole story just to find out what I'm doing." Another student, who wrote all of his equations as either “a×b=c” (if there was not an intercept term) or “a+b =c” (if there was an intercept term), justified the importance of his symbolic representations by saying that “it lets me keep track of stuff” and explaining how he could substitute his slope value in for “a” and his x-value in for “b” in the first equation. This student and two others who wrote equations that were “too general” – i.e. that used letters to represent the given parameters of slope and intercept – may have been showing early signs of structural or object-oriented thinking, in that they were considering linear equations as classes, some having intercepts and some not.

Overall, when students wrote symbolic equations to go with story scenarios, they often existed as entities that were separate from students’ work on result unknowns and start unknowns. Some students in the study showed little to no understanding of writing equations to go with story scenarios. Students’ limited understanding of symbolic representations could be interpreted as a result of poor instruction, but could also be framed as being in part due to the larger system of school mathematics where situational reasoning is not well-connected to formal mathematical representations. Personalizing a story to students’ interests and experiences did not appear to enhance students’ understanding of the relationships in the story in a way that supported the writing of equations and using equations with understanding. The rates which with students wrote correct equations and wrote equations disconnected from situation-based reasoning were similar for personalized problems compared to other problem types. There were also fewer attempts to use the formal strategy of equation solving to solve a start unknown for personalized problems compared to other problem types.

Students’ success writing stories to correspond to symbolic equations

Students given the completely abstract problem type (i.e. bare symbolic equations) were asked to write a story scenario that could correspond to the symbolic equation. Although this task has been posed to students in other studies of algebra story problems (Nathan et al., 1992; Alibali, Kao, Brown, Nathan, & Stephens, 2009; Stephens, 2003), the purpose here was different. It was of interest to analyze how the story problems written by the students would reflect their beliefs about the story problems they are given in school mathematics classes. Out of the 10 students who were asked to write a story scenario to go with an equation, 5 did not respond to the question, insisting that they did not know how to write a story to go with a symbolic expression. This was unexpected, given that the students had been observed doing a similar activity in class on two different occasions. However, it is important to consider the scaffolding that is in place in a classroom context – on both of these occasions, the teacher demonstrated to students how they could write a story to go with a symbolic equation first, and the students usually closely imitated her example, sometimes just changing the numbers. They were also working in groups and getting further assistance from the teacher as needed. Without these supports in place and a recent example to imitate, the students' understanding again seemed shallow.

The story scenarios written by the 5 students who did respond are shown in Table 9. More than anything else, this table reveals how difficult it is for students to write a story for a symbolic equation, and suggests a number of factors of difficulty that are involved in this task. First, students must be able to reason structurally (i.e. object perspective) about the equation in terms of slope and intercept parameters. Students need to understand the slope parameter as a rate of change, and the intercept term as an additive constant. The student writing the pencil scenario in the second row of the table seems to struggle with these ideas, and uses the slope term as an additive constant.

Students must also have some understanding of the concept of a variable. For instance in the first row of Table 9, the student generated a concrete value of the dependent variable (2 years), rather than expressing the relationship more generally as is done in a symbolic equation. Writing a story scenario to go with an equation may be difficult if students are not able to adopt a process conception of a function, or view a function as a systematic relationship between variables. Given that students' thinking about functions was still heavily action-oriented, the difficulty solving this problem part makes sense. Finally, students must also be able to successfully engage in unit analysis, which is a difficult skill even in college-level work. The student in the third row does not match the units of the two quantities that he's adding together (pieces of gum + dollars), and the student in the fourth row simply makes every possible unit "cookies."

Symbolic Equation	Student-generated story scenario
y=12.95x	Sam (wanted to save money), so he started at $12.95, and each month it doubled. So he wanted to see how much money he'd have in two years, which would be y.
y=80-6x (had previously solved if y=8, what is x?)	You have 80 pencils and give away 6 pencils so you have 74 left. And (you) lost 8 pencils (so) now you have 66 pencils left.
y=4x+175	Tom had gotten x, the amount of boxes, each containing 4 gum. And he had $175 and it cost blank (_____).
y=1.25x+2.5	y equals the total amount of cookies, and there is 1.25… which is one cookie and another piece of a cookie. Alright so 1.25 equals the amount of cookies, plus another 2.5, and x will be, the, x will be how many… so x would also be cookies. So what we're trying to figure out the total amount of cookies there are.
y=0.75x+10	There is a store that sells candy for $0.75, and for tax they charge $10. (Student writes "total" above y, and "# of candy" above x).

Table 9. Five students' responses to the "write story" problem part of an abstract problem

Students must coordinate their reasoning successfully along all of these dimensions before it becomes truly important to consider whether the story being written is "realistic." The story in the final row of Table 9 about candy was the only story written by a student that was coded as being intended/correct. As can be seen from this student's response, writing an appropriate story includes being able to successfully interpret the independent variable, the dependent variable, the slope, and the intercept in a coordinated manner, and in terms of an everyday situation. However, this student's story scenario certainly is not realistic – a customer would have to buy a lot of candy before a $10 tax is reasonable, and framing tax as a constant intercept value does not make sense in any case. This student and the other students in the study may have viewed story problems as stereotyped "hidden equations" that did not need to make realistic sense.

VIII. Chapter Eight: Implications

A. *Summary and Discussion*

A series of clinical interviews of Algebra I students from a low-performing high school were conducted with the intent of critically examining the efficacy story problems, the most common way of contextualizing mathematics in this school setting. As part of the study, problems on linear functions were personalized to students' interests and experiences. Although there are studies suggesting benefits for personalization, the research results are mixed and have not been conducted with algebra level story problems. Implementing such personalization was difficult; one issue was the challenge of designing scenarios where one quantity depended on another – i.e. scenarios with proportional relationships – but also choosing scenarios where a start value or intercept term would have strong situational meaning. It was also challenging for students to articulate mathematical situations from their everyday life that were not simple arithmetic scenarios. Students overwhelmingly perceived personalized problems as being easier than other problems, including normal story problems. Personalized problems also had a very low "no response" rate, and high incidence of students using informal strategies when compared to some other problem types. This suggests that personalized problems may help make math more accessible to the students who struggle most with algebra.

In terms of the common justifications for placing mathematics in context, findings showed that on the surface students largely adhered to the idea that the concepts they were learning in algebra would transfer to their everyday adult lives, and that algebra would be practically useful in the workplace for some professions. Future research could investigate how these beliefs are cultivated in the system of school mathematics, how they connect to symbol precedence views prevalent in textbooks, and how they impact students' mathematical development.

The results also showed that contextualized problems provide students with access to mathematical ideas, supporting the verbal precedence view (Nathan & Petrosino, 2003). Story problems had lower "no response" rates, with more informal strategies being used on concrete and relatable problems. However, the interviews also demonstrated that it can still be challenging for high school students to verbally interpret a story scenario. Verbal interpretation issues were most often either vocabulary issues or students' confusion over how the story described the start or intercept value.

The idea that story problems may provide students with access to mathematical ideas because students explicitly use participation structures from everyday situations when solving them found limited support; explicit use was rare, and these participation practices were disruptive or nonproductive as often as they helped. On the other hand, the prevalence of students' use of informal, situation-based reasoning and arithmetic strategies on contextualized problems suggests that students may implicitly use participation practices from everyday situations when their connection to the context is strong. However, as with some of the "disruptive" ways in which students used situational knowledge, such informal reasoning is not traditionally valued in school mathematics classrooms. There were cases where students would make reasonable inferences based on either the verbal semantics of the story or their situational knowledge, but these inferences were unintended by problem authors and led to incorrect answers. There were also instances where students should have realized an answer to a story problem was unreasonable, but chose not to take everyday knowledge into account.

These findings about students using everyday participation practices when solving story problems mirror studies of arithmetic story problems, which problematize the authenticity of story contexts within the larger system of school mathematics. Students used *non-coordinative* approaches when solving story problems, where they bypassed an intermediate understanding of the given situation before moving to solve the problem. Non-coordinative approaches were especially prevalent in the story problems that included equations; students struggled to coordinate situation and problem models in this problem framing. However, in the system of school mathematics where problem scenarios are sometimes stereotyped and oversimplified, this could be interpreted as a rational, conformist approach.

Students' use and understanding of symbolic equations written to correspond to story contexts also accentuated the effects of the larger social system of school mathematics. Students often saw these equations as disconnected from their situation-based reasoning, and rarely used symbolic equations that they had generated to solve start unknowns. Symbolism is often employed in introductory algebra classes as an end in and of itself, for purposes it may not actually be used for, and as a simple generalization of arithmetic.

Greer (1997) makes the important point that if students are going to use and understand symbolic representations, they need experience building powerful mathematical models to solve extended and complex problems. Scaffolding students towards increasingly abstract representations, like symbolism, should be developed over a long period of time, and symbols should be viewed simultaneously as both abstract mathematical tools and as concrete entities tied to situation-based meaning. However, one interesting finding of this study was that some students seemed to view a symbolic equation embedded in a story context as being more accessible than a "bare" symbolic equation with a similar structure. This is a finding that would be interesting to follow up on, as little research has been done on story problems that include equations, and these types of problems are also prevalent, especially in college mathematics.

B. Theoretical Implications

The strength of a qualitative study of this type is not simply to reach an overall theoretical conclusion that story problems are effective because they are verbal, because they elicit certain strategies, because they connect to students' everyday participation practices, or because they are motivating. This study demonstrates that the effects of contextualization are complex, with multiple factors of difficulty and facilitation operating simultaneously, each being specific to the individual cover story of the problem, the underlying mathematical structure, and the individual student and their experiences participating in in-school and out-of-school systems.

Theoretically, this study was framed with respect to a situated cognition perspective, where learning is viewed as participation in complex, social systems. A perspective that is highly related to a situated cognition view, and that perhaps allows for finer-grained interpretations of student thinking, is the manifold ontology of mind view of cognition, proposed by Hammer, Elby, Scherr, and Redish (2005). From this perspective, students have access to a myriad of fine-grained cognitive resources which are activated in sets as they solve problems. These resources are not “correct” or “incorrect” in and of themselves, however their conditions of activation can be appropriate or inappropriate with respect to the content domain. Activation of resources is tied to the features of the immediate context; the way in which students epistemologically and conceptually frame the context and situation affects the sets of resources they choose to activate or not activate. Hammer et al. (2005) describe a *frame* as “a set of expectations an individual has about the situation in which she finds herself that affect what she notices and how she thinks to act” (p. 9).

This perspective is important to the research presented here because like the situated cognition perspective, it has explanatory power for the context-dependence of reasoning. From a situated cognition perspective, students’ reasoning may be different in different contexts because their participation structures are modified by the characteristics of the complex, social systems acting upon them, and the norms, beliefs, and values of these systems. From a manifold ontology of mind perspective, students’ reasoning is tied to the cognitive resources they choose to activate based on how they frame both the immediate problem and situation, as well as how they frame learning, knowing, and activity in their larger systems of participation. Hammer et al.’s (2005) manifold ontology of mind perspective in many ways is similar to the perspective described in Greeno (1991), where knowledge in a domain is described as the ability to productively locate and activate resources. As Greeno (1991) writes:

> In the environmental view, knowing a set of concepts is not equivalent to having representations of the concepts but rather involves abilities to find and use the

concepts in constructive processes of reasoning… the person's knowledge, however, is in his or her ability to find and use the resources… (p. 175)

Results of the present study support a manifold ontology of mind perspective (Hammer et al., 2005), and suggest that students frame story problems differently based on their wording, concreteness, relevance, and structure, and this framing has implications for the resources they activate when solving the problem.

Figure 24 shows a revised version of the model of story problem solving proposed earlier based on the literature review and the pilot studies (shown in Figure 9). In this new model, rather than "rules of the story problem game" mediating problem-solving acts and decisions, the broader notion of an *epistemological frame* is employed. Here students' epistemological frames include their conceptions of how story problems "work," why they are given story problems in school, and how story problems should be solved. Epistemological frames also account for students' broader participation practices in school mathematics, their beliefs about mathematical knowledge and representations, and their conceptions of how mathematics is used to solve problems in other social systems. Students' problem solving is also mediated by *conceptual frames*. In the present study, students' conceptual frames include how they perceive the mathematical concepts like "function" and "variable," how they use representations in the context of the problem they are solving, and whether they frame story problems arithmetically or algebraically.

Functions were conceptually framed by many students in the study from an action perspective, and variables were framed as specific unknown values, activating resources related to arithmetic. Resources based in arithmetic operations allowed students to solve result unknowns with simple forward calculations, but were more problematic for start unknowns. Students' tendency to represent linear functions as strings of operations was also consistent with resource activations related to arithmetic, and framing a function as an action and a variable as a "missing number." Students sometimes inappropriately activated resources relating to directly proportional relationships, demonstrating their understanding of multiplication, rather than reasoning about a slope and intercept term simultaneously. Students had limited resource activations related to mathematical representation; representing problems graphically, in tabular form, or symbolically may not be relevant resource activations when students frame a function as an action and are focused on performing simple arithmetic operations.

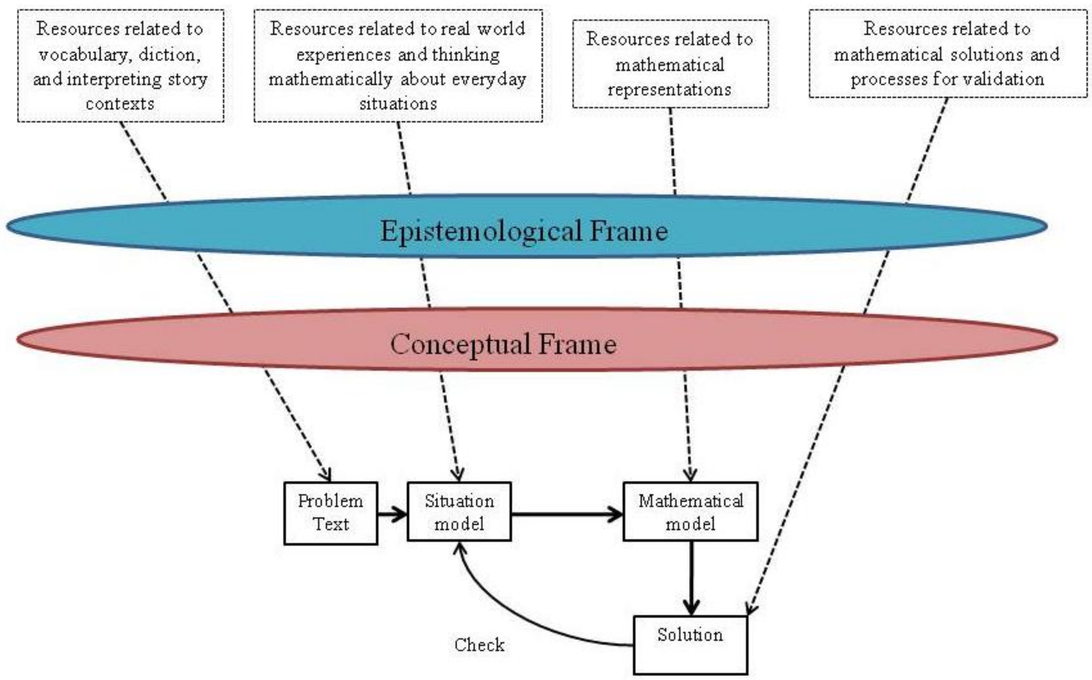


Figure 24. Revised model of story problem solving, based on framework proposed by Hammer et al. (2005) and original model in Greer (1997)

Findings also suggest that while students may activate epistemological resources affirming the relatedness of story problems to human activity when specifically asked why they are given such problems to solve in algebra class, when actually solving these problems students rarely explicitly call upon situational knowledge and often use approaches that bypass situational understanding. This suggests that students may sometimes epistemologically frame solving story problems as a meaningless, calculational activity, imbedded in a system of schooling that on the surface accentuates the applicability of mathematics, but whose practices are often unrelated to authentic uses of mathematics as a tool to solve complex, situated problems.

However, many students did activate resources based on their experience with everyday participation practices when solving story problems, through the informal, situated-based strategies they used, and occasional explicit applications of situational knowledge to story texts. This suggests that sometimes students epistemologically frame story problems as related to activity in other social systems, and attempt to activate relevant experience-based resources. However, many of these activations would not be valued in the school mathematics system, and can even lead to unintended answers. Students in this study also needed to activate complex verbal interpretation resources to form a situation model from the problem text, using their epistemological and conceptual frames to make decisions about what the problem author intended.

One of the original research purposes was to see if personalized story problems changed students' conceptual and epistemological frames by enhancing sense-making and situational reasoning; results are mixed. While personalized problems were more accessible to students, and students were more likely to activate resources related to using valid, arithmetic approaches to solve personalized problems, there was evidence that students sometimes epistemologically framed these problems as "school mathematics" tasks. It is important to remember that the personalized problems used here were by their nature somewhat removed from students' experiences. The interviewer framed the students' stories to "fit" linear functions, chose the language and vocabulary the problem would use, explicitly asked for certain representational forms, and had certain predispositions to what were "intended" interpretations and "intended" answers. Thus although related to students' interests and experiences, personalized problems were still "traditional story problems."

Overall, results suggest that research in algebra should focus not only on tasks and modes of instruction that teach students important mathematical resources, but should also focus on scaffolding students' adoption of appropriate and powerful conceptual and epistemological frames. Contextualization of mathematical ideas should be situated in the students' own lived worlds, and the diverse formal and informal mathematical sense-making activities they engage in.

C. *Implications for Curriculum, Instruction, and Assessment*

In terms of classroom application, this study points to the importance of teachers even at the algebra level explicitly addressing potentially different interpretations of stories, as well as discussing verbal reasoning about stories. These issues could have real implications for students on standardized tests, as shown in the elementary literature (Kazemi, 2002). Mathematics educators using exemplary approaches to instruction should specifically address and use traditional story problems with their students, presenting these problems as a cultural artifact, and calling attention to their specific, sometimes nonsensical, norms and expectations. Otherwise, students may become confused about the apparent disconnect between the norms of story problem solving in school mathematics and the norms of how mathematics is used as a tool in other social systems.

This study also suggests that beginning algebra instruction that is targeted to students' interests and experiences, to the ways in which they use mathematics in their everyday lives, could provide the students who struggle most access to algebraic ideas. By using progressively less concrete problem representations, students could move from informal strategies and situation-based reasoning to developing an understanding of the powerful mathematics tool of abstraction; this trajectory is in contrast with the symbol precedence organization found in many algebra textbooks (Nathan, Long, & Alibali, 2002). However, it is important to accentuate that the continuum of concrete mathematical experiences should not simply extend from "story problems" to "symbolic equations;" students need experience using mathematics to solve complex, authentic problems where they develop a true purpose and rationale for the use of representational tools. Several examples of "contextualized mathematics" that meet these criteria were discussed, including the work on model-eliciting activities (Lesh & Zawojewski, 2007) and the Algebra Project curriculum (Moses & Cobb, 2001).

The idea of personalizing mathematics problems to students' interests and experiences is becoming more feasible (and perhaps inevitable) as advances in technology provide powerful methods to individualize curriculum and instruction. This study could be considered a test of the "best case scenario" of personalizing traditional story problems – students were asked to discuss the ways in which they used mathematics in their everyday life during a semi-structured interview, and the interviewer wrote specific problems based on these discussions. While this study suggests benefits for such personalization related to performance, problem access, and students' motivational outlook on solving problems, this study also shows that when traditional story problems are used as "base problems," there are important constraints with respect to the epistemological statements made to students about applied mathematical activity.

In terms of assessment, a central argument here is that participation in a social system where tasks like traditional word problems constitute mathematical activity may not always be related to participation in systems where similar norms are not in place. This leads to the important implication that the "application" story problems that are prevalent in many high-stakes standardized mathematics tests may be assessing skills that have little to do with the knowing of algebra, and in recent years both curriculum and instruction have become increasingly aligned to these assessments. These ideas further call into question how any reform-based interventions focusing on participation in authentic mathematical activity can possibly show the standardized test gains that are required for sustainability.

D. Limitations

The set of studies reported here has several important limitations. First, 24 students from the school site were interviewed. While this is an adequate sample size for this type of in-depth qualitative analysis, certainly this line of research would only be strengthened by looking at the reasoning of many students, across a range of different settings. A follow-up study that has been conducted using algebra story problems, not discussed here, extends some of these analyses to a much larger group of students, but this increased sample size also led to other tradeoffs. Second, here only one type of story problem is examined – story problems on linear functions – although there are in fact a variety of other types of story problems that are prevalent in algebra courses.

Third, the interview setting cannot be assumed to correspond directly students' problem solving in "school mathematics" – in fact, several instances are identified throughout the study where students' reasoning was different in the interview setting than observed in the classroom, due to the fact that there were fewer resources available in the interview situation. The interviewer's significant amount of time spent in the classroom was intended to partially off-set this weakness by allowing for comparative discussion of these two contexts. And finally, in the analysis here it is unclear the degree to which readability of story problems mediates problem solving, and how this interacts with different problem framings. In future work, the readability grade level of the story problems will be included as a factor in the analysis.

E. Conclusion

Based on the results of the studies presented here, several proposals relating to the contextualization of mathematics in classrooms are now presented. First, math educators need to expand this notion of a "story problem" to include complex, open-ended scenarios where multiple solutions and interpretations are valued, or rather than seeking a "solution" at all, students engage in realistic modeling activities where mathematical structure is revealed through progressive iterations (e.g. Chazan, 1999; Lehrer & Schauble, 2006; Lesh & Zawojewski, 2007; Moses & Cobb, 2001). The traditional story problems discussed here are more appropriate to use as practice problems, once the central ideas of the applicable content objectives have been grappled with through modeling or investigation activities, and students have a clear understanding of how the concepts they are studying "work" and what the purpose of learning about these concepts is with respect to applications. As practice activities, traditional story problems themselves could be structured to become progressively more abstract, moving from personalized story problems, perhaps highly related to the modeling or investigation activities conducted by students, to normal story problems, and then to problems explicitly involving symbolic equations.

Second, these results cause concern about the prevalence of traditional story problems on standardized high-stakes assessments, and the epistemological statements these problems make to students and teachers about the knowing and doing of mathematics. As long as standardized assessments have consequences for students, teachers, and schools attached to them, they will have a powerful influence on the focus of curriculum and instruction, and the types of contextualization that are valued in school settings. It is unclear whether these assessments could ever be designed in such a way that they could assess students' understanding of modeling the world using mathematics, and still retain the types of validity that are valued and demanded by stakeholders like policy makers and test developers. If a body of research that spans across grade levels, instructional settings, and mathematical concepts can show strong evidence for the inadequacy of traditional story problems to assess what students know about modeling with mathematics, a case can be built for change. The studies reported here contribute to this body of work.

Third, these studies suggest that more research needs to focus on the idea of contextualized mathematics, seeking to gain a greater theoretical and practical understanding of how different levels and types of contextualization impact students' problem solving. It is a deeply ingrained societal belief that traditional story problems represent applications rather than contrived "hidden algorithms." The discontinuities seen in this series of studies, evidenced by students' multiple interpretations, non-coordinative approaches, and limited use of situational knowledge, need to be further grounded in empirical work on students' problem solving. Hammer et al.'s (2005) framework, designed with physics in mind, seems to be firm ground for thinking about contextualized mathematics, as it allows two essential questions to be addressed: (1) How can we design and present mathematics activities such that students acquire and use epistemological frames that correspond to current understandings of how people come to know mathematics? and (2) How can we design and present mathematical activities such that students acquire and use conceptual frames that provide access to significant mathematics learning, and push students to think about concepts in new and productive ways?

Most importantly, these studies point to a need to reform algebra instruction such that traditional story scenarios are no longer the primary manner in which school mathematics is connected to participation in systems where mathematics is used as a tool to solve real problems. A recent survey conducted for the National Mathematics Advisory Panel report found that when Algebra I teachers were asked to identify the single most challenging aspect of teaching algebra, the overwhelming response was "working with unmotivated students," and the second most frequent response was "making mathematics accessible and comprehensible to all of my students" (Loveless et al., 2008). Both of these concerns are highly related to issues of contextualization, and the choices being made by education stakeholders about how school mathematics can be framed as "relevant" to students' lives and experiences.

The research presented here demonstrates some of the implications of exposing a group of high risk students to an algebra instruction that is heavily procedural and often disconnected from participation structures valued in applied formal and informal mathematical activity. This study and the body of work surrounding story problems suggests that educational researchers and practitioners need to move beyond thinking of mathematical activities as "contextualized" or "not contextualized," and instead focus on how different *types* of contextualization mediate students' participation practices. The real story that story problems tell is not only one of verbal precedence models and efficient procedures for solutions; these problems tell us about the system in which students today are learning to reason mathematically.

It is unlikely that story problems will leave algebra class any time soon; they are prevalent in mathematics curriculum, instruction, and assessment from kindergarten to college (Jonassen, 2003) and mathematics word problems have been used in human society for over 5000 years, dating back to the fourth millennium B.C. in ancient Mesopotamia (Swetz, 2009). However, it is important that teachers, curriculum developers, assessment designers, and researchers in mathematics education understand both the affordances and constraints of what is defined in this paper as "traditional story problems," and proceed with caution and thoughtfulness when integrating these problems into the system of school mathematics.

IX. Epilogue: Students' Standardized Test Scores

Although the work presented here is qualitative in nature, discussions with colleagues led to requests for a more in-depth analysis of the standardized test scores of students in the study. In the state of Texas, standardized testing drives much of what schools do, particularly in low-performing schools like the one described in this research. Little is known about how the problem-solving behaviors identified here, such as use of non-coordinative approaches, use of situational knowledge, and issues with verbal interpretation mediate students' actions and affect performance on standardized exams. Since the test scores of the students in the study were relatively easy to obtain, I decided to engage in a follow-up analysis of how standardized exam scores differed systematically between students who had different profiles of problem-solving behaviors during the interviews. Since there is no way to determine what actual problem-solving behaviors the students used while being tested, this analysis can only be suggestive.

The standardized test scores in mathematics for the students participating in the study were collected after the study had concluded, and in this chapter a quantitative analysis of these results is provided. In Texas during the study, the standardized test was referred to as the "Texas Assessment of Knowledge and Skills" (TAKS). During the year of the study, the state standardized test for ninth grade mathematics contained 52 problems, 23 of which were framed as story problems and 24 of which related directly to functions. The scaled score needed to meet the minimum standard in mathematics was 2100, with a score of 2400 or above being "Commended" performance. The average mathematics TAKS score for the 24 students in the study was 2131 (standard deviation of 290); the highest score was 2636 and the lowest was 1063. Nine of the 24 students (37.5%) did not meet the minimum standard in mathematics, 13 (54.2%) met the minimum standard but were not commended, and 2 (8.3%) achieved commended performance. These were similar to the percentages for all ninth graders at the school site.

Subsets of the 24 students in the study were compared along various dimensions identified from the qualitative analysis, looking for significant differences in TAKS scores in mathematics. The district would not provide individual student scores without parental consent; they instead provided averages and standard deviations for each set of subgroups. The data was first analyzed by comparing the math TAKS scores of students who were successful (50% or more intended answers) at the interview versus students who were unsuccessful (under 50% intended answers) at the interview.

The analysis then looked for differences in TAKS scores between students who used non-coordinative approaches more than twice versus those who had 0-2 instances of non-coordinative reasoning. Differences in TAKS scores were also examined for students with 5 or more issues with verbal interpretation, versus those who had less than 5 instances. These cut-off values were determined by looking for divisions in the data that allowed a reasonable number of students to be in each subgroup, and such that there would be a minimum number of students who were "borderline."

TAKS scores were also compared for students who explicitly used everyday participation practices productively to solve problems or who were able to successfully interpret parameters in a normal with equation problem type, versus those who did not do either of these things. Finally, TAKS scores were compared for students who used informal strategies like repeated addition and trial and error to solve start unknown problems versus those who did not, and the scores of 5 students who successfully used equation solving to solve a start unknown were compared to the rest of the student scores. Due to small subgroup sample sizes and very high variances in test scores (note again that the highest score was 2636 and the lowest was 1063), no differences between groups reached statistical significance at the 5% level after a Bonferonni correction for repeated t-tests had been applied. Table 10 summarizes the results of the t-tests conducted.

The first column of the table describes the two groups of students being compared by the t-test, while the second column gives the alternate hypothesis for the relationship between the two groups. The third column gives the p-value of the t-test that was performed and whether the difference between the groups was significant. The fourth column notes whether the difference was in the hypothesized direction, while the fifth and final column gives the effect size, measured by Cohen's *d*.

Effect size measures the magnitude of the effect of a treatment, and Cohen's *d* is calculated by taking the ratio of the difference in the means of the two groups to their pooled standard deviation. A Cohen's *d* of 0.2 is tentatively considered to be a small effect, 0.5 a medium effect, and 0.8 a large effect (Cohen, 1988). Here there is no sense of a "treatment" in the traditional sense, and the effect size values can simply be interpreted as how many standard deviations the means of the two groups differ by. Students who did poorly on the interview questions and students who used informal strategies to solve problems tended to have lower math TAKS scores, and the effect sizes were medium to large (1.23 and 0.78 respectively).

Description of two groups being tested	**Alternate Hypothesis**	**p-value**	**Expected direction?**	**Effect Size (*d*)**
Students who were successful (50%+ intended answers) at the interview problems versus students who were not successful (under 50% intended answers) at the interview problems	Successful students > Unsuccessful students	0.035 (NS)	yes	1.23
Students with few (0-2) instances of non-coordinative reasoning versus students with more (3+) instances of non-coordinative reasoning	Not use non-coordinative> Use non-coordinative	0.631 (NS)	yes	0.21
Students with many (5+) issues with verbal interpretation, versus students with fewer (0-4) issues with verbal interpretation	Many verbal issues < few verbal issues	0.405 (NS)	no	0.32
Students who used situational knowledge productively versus other students	Use situational knowl > other students	0.257 (NS)	no	0.67
Students who used lowest level informal strategies (repeated addition and trial and error) versus other students	Informal strategy students < other students	0.110 (NS)	yes	0.78
Students who used equation solving successfully versus students who did not use or did not use successfully	Equation solving students > other students	0.458 (NS)	yes	0.25
The p-value being used to determine significance was p = .05/6 = .0083 *NS = Not significant*				

Table 10. Summary of significance of results for Welch's t-tests performed on mathematics scaled TAKS test scores for subgroups of 24 students in sample

In two cases, the relationships did not go in the hypothesized directions. Students who used situational knowledge productively to solve problems or interpret parameters during the interview actually scored lower in terms of mathematics TAKS scores than those who did not. Although this difference did not reach significance, the effect size was medium (0.67). This may be an idiosyncrasy of a small data set with high variance values, or it may suggest that a tendency to use everyday participation practices on TAKS-style story problems is not a particularly useful skill.

Students who had significant issues with verbal interpretation during the interview actually scored higher than those who did not in mathematics (although they scored lower in reading), however this difference did not reach significance and had only a small effect size (0.32). This again could be an idiosyncrasy of the data set, or it may suggest that being able to read and interpret situational contexts in standardized test-style problems is not especially helpful. Thinking back to Tammy and her floor plan (Introduction), one could see how skills like using situational knowledge and being able to read and understand a story situation may not be important or useful on standardized test questions.

The reading standardized test scores for the sample of 24 students were also requested. The average reading scaled score for the sample was 2214 (standard deviation of 147); again, a score of 2100 was needed to meet the minimum standard, and a score of 2400 or above was “Commended.” The lowest score in the sample was 1863 and the highest was 2481. Five of the 24 students (20.8%) did not meet the minimum standard in reading, 16 (66.7%) met the minimum standard but were not commended, and 3 (12.5%) were commended. These were similar to the percentages for all ninth graders at the school site.

Only two comparisons were of interest for the TAKS reading scores - differences between students who were successful versus unsuccessful at the interview, and differences between students who had many (5+) issues with verbal interpretation of stories versus those who had fewer. The results, summarized in Table 11, again did not reach statistical significance. However, performing poorly on the interview problems was associated with lower reading TAKS scores, with a large effect size (0.83).

Description of two groups being tested	**Alternate Hypothesis**	**p-value**	**Expected direction?**	**Effect Size (*d*)**
Students who were successful (50%+ intended answers) at the interview problems versus students who were not successful (under 50% intended answers) at the interview problems	Successful students > Unsuccessful students	0.112 (NS)	yes	0.83
Students with many (5+) issues with verbal interpretation, versus students with fewer (0-4) issues	Many verbal issues < few verbal issues	0.594 (NS)	yes	0.25
The p-value being used to determine significance was p = .05/2 = .025 *NS = Not significant*				

Table 11. Summary of significance of results for Welch's t-tests performed on reading scaled TAKS test scores for subgroups of 24 students in sample

Appendix A – Student Entrance Interview

1) How do you use math in your everyday life?
2) When do you use numbers outside of school?
3) Why do you think you're learning algebra in 9^{th} grade?
4) How do you think algebra will be useful to you in the future? What about outside of school and college, what about in real life?
5) What kinds of things do you like to do when you get home from school?
6) What do you like to do on the weekends?
7) What kinds of things did you do over your last summer break?
8) Do you have any hobbies? Can you tell me about them?
9) **Sports** – Do you play any sports? How long have you been playing for? Do you like to watch any sports?
10) **Movies** – Do you like watching movies? What kind of movies do you like? Where do you get movies from or watch movies?
11) **Video games** – Do you like to play video games? What game systems are your favorites? What games do you like? Are there any numbers in the games you play?
12) **T.V. Shows** – Do you like watching T.V.? What channels do you watch? What shows do you watch? How much do you watch T.V.?
13) **Shopping** – Do you like to go shopping? What kinds of things do you like to shop for? What are your favorite stores? Do you look for sales? What kind of sales? Do you ever buy stuff online?
14) **Computers** – Do you spend any time outside of school on computers? What do you like to do on the computer? Do you play games on the computer? What games? Do you use MySpace? Do you download stuff? Do you see numbers when you're on the computer?
15) **Food/Cooking** – What are your favorite foods? What restaurants do you like? Do you like to cook? What dishes do you like to cook? Do you have to know any math to do cooking stuff?
16) **Music** – What kind of music do you listen to? What do you listen to your music on? (radio, Ipod, etc.) Where do you get your songs from? How much are they?
17) **Phone** – Do you like to talk on your phone? Who do you like to talk to? How much do you use your phone every day? How much do you text on your phone? Do you ever see your bill or how many minutes you use?
18) **After-School Clubs** – Are you in any clubs or organizations that meet after school? Tell me about them. Why do you like this club?
19) Do you work at a job? Do you drive? How do you need to do math in these situations if so?

Appendix B – Base Problems for Full Interview

Num	Problem
1	During the last school year, 25% of the total students enrolled in Algebra earned an A or B. a) If 440 students were enrolled in Algebra last year, how many earned an A or B? b) If 700 students were enrolled in Algebra last year, how many earned an A or B? c) Write an algebra rule that represents this situation using symbols. d) If 40 students earned an A or B in Algebra last year, how many total students were enrolled?
2	The world's fastest passenger jet has a cruising speed of 1500 mph. Suppose this jet has already flown 500 miles from New York towards London. a) How far from New York will the jet be 1.5 hours from now? b) How far from New York will the jet be 30 minutes from now? c) Write an algebra rule that represents this situation using symbols. d) How long before the jet arrives in London? (Recall that the distance between London and New York is 3500 miles.)
3	A mail order company charges a flat fee of $10 for shipping and handling per order, independent of the order's cost. The company is currently running a sale where they are discounting each item in the order by 25%. a) If items total $40, how much would this order cost during this sale? b) How much would this order cost during this sale if, before the sale, the items cost $120? c) Write an algebra rule that represents this situation using symbols. d) If the cost of an order during this sale is $85 what would the cost of the items be without the sale?
4	Some early Native Americans used clam shells called Wampum as a form of currency. Tagawininto, a Native American, had 80 wampum shells, and spends 6 of them every day. a) How many shells did Tagawininto have after 10 days? b) How many shells did he have after a week? c) Write an algebra rule that represents this situation using symbols. d) After how many days did he have 8 shells?
5	Due to a billing error last month, Amanda has received a $7.87 credit towards next month's cellular phone bill. She pays a flat $0.23 per minute with no additional monthly charge. a) If 43 minutes are used, calculate the bill for the next month. b) If 100 minutes are used, calculate the bill for the next month. c) Write an algebra rule that represents this situation using symbols. d) After finding that billing error last month, this month Amanda will make sure that her bill is correct. If her bill is for $38.13, how many minutes has she talked?
6	A skier noticed that he can complete a run in about 30 minutes (half an hour). A run consists of riding the ski lift up the hill and skiing back down. a) If he skis for 3 hours, how many runs will he have completed? b) If he skis for 6 hours, how many runs will he have completed?

	c) Write an algebra rule that represents this situation using symbols. d) If he plans on making 10 runs, how many hours will he have to ski?
7	A machine called the Crawler, which moves space shuttles, travels at the rate of 4 feet per second. The Crawler is currently 175 feet from the hanger, moving toward the launching pad. a) How far will the Crawler be from the hanger in 20 more seconds? b) How far will the Crawler be from the hanger in 1 more minute? c) Write an algebra rule that represents this situation using symbols. d) In how many more seconds will the Crawler reach the launching pad, which is a total of 275 feet from the hanger?
8	Wall-to-wall carpet is sold by the square yard. One type of carpet costs $12.95 per square yard. a) How much would 12 square yards of this carpet cost? b) How much would forty one square yards of this carpet cost? c) Write an algebra rule representing this situation using symbols. d) How much of this carpet could be installed for a cost of $388.50?
9	Some rental cars have mobile phones installed. In one car, the cost of making a call from the mobile telephone is $1.25 per minute with an initial fee of $2.50. a) How much would a call that lasted ten minutes cost? b) How much would a call that lasted five minutes cost? c) Write an algebra rule representing this situation using symbols. d) If a call cost a total of twenty dollars, how many minutes did the call last?
10	An experimental liquid (LOT#XLHS-240) is being tested to determine its behavior under extremely low temperatures. Its current temperature is -35 degrees Celsius and is slowly being lowered by two and one-half degrees per hour. a) What will the temperature of the liquid be ten hours from now? b) What will the temperature of the liquid be tomorrow at this time? c) Write an algebra rule representing this situation using symbols d) Assuming that the temperature has been dropping at the same rate, when was the temperature zero degrees Celsius?
11	A huge mirror for a telescope is being moved by a truck with 13 axles and 50 tires, from Erie, Pennsylvania to Raleigh, North Carolina. The truck averages 15 miles per hour and has already traveled 60 miles. a) In three more hours, how many total miles will the truck have traveled? b) How many total miles will the truck have traveled in eight more hours? c) Write an algebra rule representing this situation using symbols. d) If the truck has been driven a total of 225 miles, how many more hours has it been driven?
12	A company has been created to produce a new product. The company predicts that its capital expenditure (the one-time start up costs to buy supplies, equipment etc.) will be $500. It plans to sell its new product for a profit of $10 per unit. (The profit per unit is the price at which it sells each unit minus the costs to make and sell each unit.) The company's profits for its first year of operation will be its total profits from sales minus its capital expenditure. e. If the company sells 190 units during the first year, how much total profit will the company make? f. How much total profit will the company make if it sells one hundred and fifty units?

	g. Write an algebraic expression for profit as a function of the number of units sold. h. How many units will the company have to sell to break even?
13	An international team of explorers plans to attempt the longest surface crossing of the Arctic Ocean in a single season. They hope to leave Russia in March and reach Canada by July - nearly three months later. To complete the 1,800 mile trip, they must average twenty miles per day, traveling in dogsleds and special canoes designed for ice-choked waters. a) How far will they travel in five days? b) After one week and six days, how far will they have traveled? c) Write an algebra rule representing this situation using symbols. d) If the explorers have traveled a total distance of 600 miles, how long have they been traveling?

Appendix C – Teacher Interview Questions

1) How do you use math in your everyday life, when you're not teaching?
2) Where do you see and have to deal with numbers outside of school?
3) Do you ever use concepts from algebra in your everyday life?
4) Why do you think students have to learn about algebra in the 8^{th} or 9^{th} grades? How will it be useful to them?
5) Solve these two problems:

A. *A mail order company charges a flat fee of $10 for shipping and handling per order, independent of the order's cost. The company is currently running a sale where they are discounting each item in the order by 25%.*
   a) *If items total $40, how much would this order cost during this sale?*
   b) *How much would this order cost during this sale if, before the sale, the items cost $120?*
   c) *Write an algebra rule that represents this situation using symbols.*
   d) *If the cost of an order during this sale is $85 what would the cost of the items be without the sale?*

B. *Due to a billing error last month, Amanda has received a $7.87 credit towards next month's cellular phone bill. She pays a flat $0.23 per minute with no additional monthly charge.*
   a) *If 43 minutes are used, calculate the bill for the next month.*
   b) *If 100 minutes are used, calculate the bill for the next month.*
   c) *Write an algebra rule that represents this situation using symbols.*
   d) *After finding that billing error last month, this month Amanda will make sure that her bill is correct. If her bill is for $38.13, how many minutes has she talked?*

6) What do you think your students would find difficult about problem A? What do you think your students would find difficult about problem B?
7) What proportion of your students do you think could successfully solve these problems? Why?
8) Which part of the problems do you think the students would find most difficult (a, b, c, or d). Why?
9) In part c) of each problem the question asks the students to construct a symbolic representation. What do you think is the purpose of representing the situation this way?
10) What do you think is the purpose of students learning about story problems like these in algebra? How often do you use story problems in your teaching? Why?

Appendix D – Coding Criteria for Interview Study Transcriptions

<table>
<tr><th>Coding Category</th><th>Possible Codes</th><th>Criteria</th></tr>
<tr><td>Outcome of Problem Part</td><td>Intended Answer
Unintended Answer
No response</td><td>- An answer was classified as “No Response” if the student did not verbally say or write an answer during the problem block
- If the student began to work a problem part (i.e. did more than just read it), but then decided to move on without solving it, it was coded as “No Response”
-Answers were coded as intended if they were the responses expected by the interviewers and the Cognitive Tutor curriculum
-Accurate symbolic equations were coded as intended answers even if they were not fully simplified, or if they did not contain a dependent variable</td></tr>
<tr><td>Result Unknown Strategies</td><td>Use Arithmetic
Use Symbolic Equation</td><td>- A result unknown strategy was classified as “arithmetic” if the students simply pulled numbers from the story and performed arithmetic operations on them
- A result unknown strategy was classified as “symbolic equation” if the student explicitly wrote out and then used a symbolic equation to solve a result unknown, or if the student used a symbolic equation given in the problem (for story with equation and abstract problem types)</td></tr>
<tr><td>Start Unknown Strategies</td><td>Trial and Error
Unwind
Solve Equation
Proportional Reasoning
Repeated Addition
Other</td><td>- A strategy was classified as trial and error if the student plugged in different x-values attempting to get the given y-value, including if they did this in table format, and including if they by chance got it on their first try.
- A strategy was classified as unwind if students arithmetically reversed the slope and/or intercept
- A strategy was classified as solve equation if the student was explicitly performing the same operations on either side of an equation to isolate x
- A strategy was classified as proportional reasoning if students multiplicatively “scaled up” from one x-value to the next, scaling up their y-values proportionally, trying to get the given y-value. Proportional reasoning also included reasoning about percent problems proportionally to solve start unknowns.
- A strategy was classified as repeated addition if the student repeatedly added the slope value, trying to get up to the given y-value
- A strategy was classified as other if it did not fit any of the above descriptions</td></tr>
<tr><td>Result Unknown and Start Unknown Mistakes</td><td>Arithmetic mistake
Forgot slope
Forgot intercept
Mixed up slope and intercept
Mixed up result unknown and start unknown</td><td>- A mistake was an arithmetic mistake if the student made a simple calculation error
- If the student didn’t include the given slope term or the intercept term in their calculation, their error was coded as forgot slope or forgot intercept
- If the student mixed up which term was the slope and/or which was the intercept, the error was coded as mixed up slope and intercept</td></tr>
</table>

	Took into account movement *Applied invalid proportional thinking* *Other*	- If the student solve a result unknown like it was a start unknown, or a start unknown like it was a result unknown, the mistake is coded as mixed up RU/SU - If the student took into account the x-value from a previously problem part in the belief that the starting point of the problem has changed, the error is coded as took into account movement - If a student used proportional thinking inappropriately (i.e. scale up a slope with the intercept value, rather than only adding on the intercept once), it is coded as applied invalid proportional thinking - Mistakes that do not fit into any of these categories are coded as other
Write Equation Mistakes	*Too general* *Too specific* *Inverted operation(s)* *No independent variable* *Mix up slope and intercept* *Forgot intercept* *Other*	- A symbolic equation is coded as too general if the parameters (i.e. slope and intercept terms) are represented by letters rather than numbers - A symbolic equation is coded as too specific if the independent and/or dependent variables are numbers instead of letters - A symbolic equation is coded as "inverted operations" if the multiplication of the slope with the intercept is written as division, or if the addition or subtraction of the intercept is inverted - A symbolic equation is coded as no independent variable if there is no independent variable in the equation, and no concrete value has been plugged in for the independent variable - A symbolic equation is coded as mix up slope and intercept if the intercept value is being used as the slope, and/or the slope value is being used as the intercept - A symbolic equation is coded as forgot intercept if it does not contain an intercept term - A mistake with a symbolic equation is coded as other if it does not fit any of the aforementioned mistakes
Use of Non-Coordinative Reasoning	*Present* *Not Present*	- A problem-solving block is coded as containing non-coordinative reasoning if the student is randomly plugging in numbers from the problem, seeking an answer that "looks right" - A block is coded as non-coordinative if the student applied a well-known schema to a problem that reading it would reveal it does not fit that schema - Also includes the case where the student simply guesses rather than trying to solve the problem (overlaps with first point)
Issue with Verbal Interpretation	*Present* *Not Present*	- This category is present if the student verbalizes that they do not understand the verbal semantics of the story, including when they directly ask interviewer for clarification -Also present if the student verbalizes an assumption about the verbal semantics of the story that is false - ONLY coded for actual verbalizations by students, NOT for assumed misinterpretations based on student work or strategies
Use of participation practices from	*Productive use* *Unproductive/ Disruptive use*	- An activation of situational knowledge is present if the student specifically verbalizes something not directly given in the story based on their everyday knowledge

everyday situations	*No explicit use*	- An activation is coded as productive if the inferences being made with respect to the everyday situation are also true with respect to the inferences needed to solve the story problem; the inferences must be mathematically relevant and have the potential to assist the student in solving the problem - An activation is coded as unproductive/disruptive if the inferences being made with respect to the everyday situation are not related to the inferences needed to solve the story problem, or are contradictory to the inferences needed to solve the story problem. - This category is not coded for “write story” and “interpret parameters” blocks – it is considered implicit (always present) in these blocks
Symbolic Equation Disconnected	*Present* *Not present*	- A symbolic equation is coded as “symbolic equation disconnected” if the equation does not match what the student previously did to solve the result unknowns - If the equation matches how they solved one result unknown, but not the other, this category is NOT present. It has to be disconnected from both.

References

Alibali, M., Kao, Y., Brown, A., Nathan, M., & Stephens, A. (2009). Middle school students' conceptual understanding of equations: Evidence from writing story problems. WCER Working Paper No. 2009-3.

American Association for the Advancement of the Sciences (2000). AAAS Project 2061 Algebra Textbooks Evaluation. Retrieved June 2009 from http://www.project2061.org/publications/textbook/algebra/summary/ia_cOver.pdf

Anad, P., & Ross, S. (1987). Using computer-assisted instruction to personalize arithmetic materials for elementary school children. *Journal of Educational Psychology*, **79**(1), 72-78.

Anderson, J. (1993). *Rules of the Mind*. Hillsdale, N.J.: Lawrence Erlbaum Associates.

Anderson, J., Corbett, A., Koedinger, K., & Pelletier, R. (1995). Cognitive tutors: Lessons learned. *Journal of the Learning Sciences*, **4**(2), 167-207.

Anfara, V., Brown, K., & Mangione, T. (2002). Qualitative analysis on stage: Making the research process more public. *Educational Researcher*, **31**(7), 28-38.

Arcavi, A. (1994). Symbol sense: Informal sense-making in formal mathematics. *For the Learning of Mathematics*, **14**(3), 24-35.

Au, K. (1980). Participation structures in reading lessons with Hawaiian children: Analysis of a culturally appropriate instructional event. *Anthropology & Education Quarterly*, **11**(2), 91-115.

Baker, R., Corbett, A., & Koedinger, K. (2004). Detecting student misuse of Intelligent Tutoring Systems. In *Proceedings of the 7th International Conference on Intelligent Tutoring Systems*, (pp. 531-540).

Baranes, R., Perry, M., and Stigler, J. (1989). Activation of real-world knowledge in the solution of word problems. *Cognition and Instruction*, **6**, 287–318.

Boaler, J. (1994). When do girls prefer football to fashion? An analysis of female underachievement in relation to 'realistic' mathematics contexts. *British Education Research Journal*, **20**(5), 551-564.

Boaler, J. (1998). Open and closed mathematics: Student experiences and understandings. *Journal of Research in Mathematics Education*, **29**(1), 41-62.

Boaler, J. (2002). *Experiencing school mathematics: Traditional and reform approaches to teaching and their impact on student achievement*. Mahwah, New Jersey: Lawrence Erlbaum Associates.

Bransford, J., Brophy, S., & Williams, S. (2000). When computer technologies meet the learning science" Issues and opportunities. Journal *of Applied Developmental Psychology*, **21**(1), 59-84.

Breidenbach, D., Dubinsky, E., Hawks, J., & Nichols, D. (1992). Development of the process conception of function. *Educational Studies in Mathematics*, **23**(3), 247-285.

Brown, J., Collins, A., & Dugid, P. (1989). Situated cognition and the culture of learning. *Educational Researcher*, **18**(1), 32-42.

Carpenter, T., Matthews, W., Lindquist, M., & Silver, E. (1983). Achievement in mathematics: Results from the National Assessment. *The Elementary School Journal*, **84**(5), 484-495.

Carpenter, T., & Moser, J. The acquisition of addition and subtraction concepts in grades one through three. *Journal for Research in Mathematics Education*, **15**(3), 179-202.

Carpenter, T., Fennema, E., Franke, M., Levi, L., & Empson, S. (1999). *Children's Mathematics: Cognitively Guided Instruction*. Portsmouth, NH, Heinemann.

Chazan, D. (1999). On teachers' mathematical knowledge and student exploration: A personal story about teaching a technologically supported approach to school algebra. *International Journal of Computers for Mathematical Learning*, **4**, 121-149.

Chi, M., & VanLehn, K. (1991). The content of physics self-explanations. *Journal of the Learning Sciences*, **1**(1), 69-105.

Clark, R., & Mayer, R. (2003). *E-Learning and the Science of Instruction*. Jossey-Bass/Pfeiffer.

Clement, J. (1982). Algebra word problem solutions: Thought processes underlying a common misconception. *Journal for Research in Mathematics Education*, **13**(1), 16-30.

Cobb, P., & Bowers, J. (1999). Cognitive and situated learning perspectives in theory and practice. *Educational Researcher,* **28**(2), 4-15.

Cobb, P., Stephan, M., McClain, K., & Gravemeijer, K. (2001). Participating in classroom mathematical practices. *Journal of the Learning Sciences*, **10**(1&2), 113-163.

Cobb, P., & Hodge, L. (2002). A relational perspective on issues of cultural diversity and equity as they play out in the mathematics classroom. Mathematical Thinking and Learning, **4**(2&3), 249-284.

Cognition and Technology Group at Vanderbilt. (1990). Anchored instruction and its relationship to situated cognition. *Educational Researcher*, **19**(5), 2-10.

Cognition and Technology Group at Vanderbilt (1997). *The Jasper Project: Lessons in curriculum, instruction, assessment, and professional development.* Mahwah, NJ: Lawrence Erlbaum Associates, Inc.

Cohen, J. (1988). *Statistical Power Analysis for the Behavioral Sciences* (2nd ed.). Hillsdale, NJ: Lawrence Earlbaum Associates.

Collins, A., & Halverson, R. (2009). *Rethinking Education in the Age of Technology: The Digital Revolution and Schooling in America*. New York: Teachers College Press.

Common Core State Standards Initiative. (2010). *Common Core State Standards (Mathematics Standards)*. Retrieved 6 July 2010 from http://www.corestandards.org/the-standards/mathematics.

Cooper, B., & Harries, T. (2009). Realistic contexts, mathematics assessment, and social class. In B. Greer, L. Verschaffel, W. Van Dooren, & S. Mukhopadhyay (eds.) *Word and Worlds: Modelling Verbal Descriptions of Situations*. Rotterdam, the Netherlands: Sense Publishers.

Cordova, D., & Lepper, M. (1996). Intrinsic motivation and the process of learning:

Beneficial effects of contextualization, personalization, and choice. *Journal of Educational Psychology*, **88**(4), 715-730.

Cuban, L., Kirkpatrick, H., & Peck, C. (2001). High access and low use of technologies in high school classrooms: Explaining an apparent paradox. *American Educational Research Journal*, **38(**4), 813-834.

Cummins, D., Kintsch,W., Reusser, K., & Weimer, R. (1988). The role of understanding in solving word problems. *Cognitive Psychology*, **20**, 405–438.

Davis-Dorsey, J., Ross, S., & Morrison, G. (1991). The role of rewording and context personalization in the solving of mathematical word problems. *Journal of Educational Psychology*, **83**(1), 61-68.

DiBiano, C., & Sabouri, P. (2006). The Implementation of Cognitive Tutor Algebra in AISD. Unpublished report from *Knowing and Learning* graduate course at University of Texas at Austin.

Ericsson, K., & Simon, H. (1998) How to study thinking in everyday life: Contrasting think-aloud protocols with descriptions and explanations of thinking. *Mind, Culture, and Activity*, **5**(3), 178-186.

Frankenstein (2009). Realistic contexts, mathematics assessment, and social class. In B. Greer, L. Verschaffel, W. Van Dooren, & S. Mukhopadhyay (eds.) *Word and Worlds: Modelling Verbal Descriptions of Situations*. Rotterdam, the Netherlands: Sense Publishers.

Gellert, U., & Jablonka, E. (2009). 'I am not talking about reality': Word problems and the intricacies of producing legitimate text. In B. Greer, L. Verschaffel, W. Van Dooren, & S. Mukhopadhyay (eds.) *Word and Worlds: Modelling Verbal Descriptions of Situations*. Rotterdam, the Netherlands: Sense Publishers.

Gerofsky, S. (2009). Genre, simulacra, impossible exchange, and the real: How postmodern theory problematises word problems. In B. Greer, L. Verschaffel, W. Van Dooren, & S. Mukhopadhyay (eds.) *Word and Worlds: Modelling Verbal Descriptions of Situations*. Rotterdam, the Netherlands: Sense Publishers.

Glaser, B., & Strauss, A. (1967). *The Discovery of Grounded Theory: Strategies for Qualitative Research*. New Brunswick: Aldine Transaction.

Gluck, K. (1999). *Eye movements and algebra tutoring*. Unpublished doctoral dissertation. Carnegie Mellon University.

Goldstone, R., & Son, J. (2005). The transfer of scientific principles using concrete and idealized simulations. *Journal of the Learning Sciences*, **14**(1), 69-110.

Greeno, J. (1991). Number sense as situated knowing in a conceptual domain. *Journal for Research in Mathematics Education*, **22**(3), 170-218.

Greeno, J. (1997). On claims that answer the wrong questions. *Educational Researcher*, **26**(1), 5-17.

Greeno, J. (2006). Learning in activity. In R. K. Sawyer (ed.) *The Cambridge Handbook of the Learning Sciences* (pp.79-96). St. Louis, Cambridge University Press.

Greeno, J., & Hall, R. (1997). Practicing representation: Learning with and about representational forms. *Phi Delta Kappan*, **78**(5), 361-367.

Greeno, J., & the MMAP Group (1998). The situativity of knowing, learning, and research. *American Psychologist*, **53**, 5–26.

Greer, B. (1997) Modelling reality in mathematics classrooms: The case of word problems. *Learning and Instruction*, **7**(4), 293-307.
Hall, R., Kibler, D., Wenger, E., & Truxaw, C. (1989). Exploring the episodic structure of algebra story problem solving. *Cognition and Instruction*, **6**(3), 223-283.
Hammer, D., Elby, A., Scherr, R., & Redish, E. (2005). Resources, framing, and transfer. In J. P. Mestre (Ed.) *Transfer of learning from a modern multidisciplinary perspective* (pp. 89-119). Greenwich, CT, Information Age Publishing.
Heath, S. B. (1982). Questioning at home and at school: A comparative study. In G. Spindler (Ed.), *Doing ethnography: Educational anthropology in action* (pp. 102-131). New York: Holt, Rinehart, & Winston.
Heffernan, N. (2001). Intelligent tutoring systems have forgotten the tutor: Adding a cognitive model of human tutors. Dissertation Proposal. Carnegie Mellon University, Pittsburgh, PA.
Hegarty, M., Mayer, R., & Monk, C. (1995). Comprehension of arithmetic word problems: A comparison of successful and unsuccessful problem solvers. *Journal of Educational Psychology*, **87**(1), 18-32.
Hinsley, D., Hayes, J., & Simon, H. (1977). From words to equations: Meaning and representation in algebra word problems. In P.A. Carpenter & M.A. Just (eds.), *Cognitive Processes in Comprehension* (pp. 89-106). Hillsdale, NJ, Lawrence Erlbaum Associates, Inc.
Humberstone, J., & Reeve, R. (2008). Profiles of algebraic competence. *Learning and Instruction*, **18**(4), 354-367.
Inoue, N. (2005). The realistic reasons behind unrealistic solutions: The role of interpretive activity in word problem solving. *Learning and Instruction*, **15**(1), 69-83.
Inoue, N. (2009).The issue of reality in word problem solving. In B. Greer, L. Verschaffel, W. Van Dooren, & S. Mukhopadhyay (eds.) *Word and Worlds: Modelling Verbal Descriptions of Situations*. Rotterdam, the Netherlands: Sense Publishers.
Jonassen, D. (2003). Designing research-based instruction for story problems. *Educational Psychology Review*, **15**(3), 267-296.
Kazemi, E. (2002). Exploring test performance in mathematics: The questions children's answers raise. *Journal of Mathematical Behavior*, **21**(2), 203-224.
Khisty, L. (1995). Masking inequality: Issues of language and meanings in mathematics teaching with Hispanic students. In Secada, W.G., Fennema, E., & Adajian, L.B. (Eds) *New Directions for Equity in Mathematics Education* (pp. 279-297), Cambridge: Cambridge University Press.
Kintsch, W., & Greeno, J. (1985). Understanding and solving arithmetic word problems. *Psychological Review*, **92**(1), 109-129.
Koedinger, K., & Sueker, E. (1996). PAT goes to college: Evaluating a cognitive tutor for developmental mathematics. *International conference on Learning sciences*, Evanston, Illinois, Carnegie Mellon University.
Koedinger, K., Anderson, J., Hadley, W., & Mark, M. (1997). Intelligent tutoring goes to the big city. *International Journal of Artificial Intelligent in Education*, **8**: 30-43.

Koedinger, K. (2001). Cognitive Tutors as modeling tool and instructional model. In Forbus, K., & Feltovich, P. (Eds.) *Smart Machines in Education: The Coming Revolution in Educational Technology,* (pp. 145-168). Menlo Park, CA: AAAI/MIT Press.

Koedinger, K., & Nathan, M. (2004). The real story behind story problems: Effects of representations on quantitative reasoning. *Journal of the Learning Sciences*, **13**(2), 129-164.

Koedinger, K., Alibali, M., & Nathan, M. (2008). Trade-offs between grounded and abstract representations: Evidence from algebra problem solving. *Cognitive Science*, **32**, 366-397.

Kvale, S. (1996) *InterViews: An Introduction to Qualitative Research Interviewing*. Thousand Oaks: Sage Publications.

Ladsen-Billings (1995). Making mathematics meaningful in multicultural contexts. In W. Secada (Ed.), *For Equity in Mathematics Education* (pp. 126-145). Cambridge University Press.

Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. *American Educational Research Journal*, **27**(1), 29-63.

Landis, J. R., & Koch, G. G. (1977). The measurement of observer agreement for categorical data. *Biometrics,* **33**, 159-174.

Lave, J. (1992). Word problems: A microcosm of theories of learning. In P. Light & G. Butterworth (Eds.), *Context and cognition: Ways of learning and knowing* (pp. 74-186). Hillsdale, NJ, Lawrence Erlbaum Associates, Inc.

Lave, J., & Wenger, E. (1991). *Situated Learning: Legitimate Peripheral Participation*. New York: Cambridge University Press.

Lehrer, R., & Schauble, L. (2000). Inventing data structures for representational purposes: Elementary grade students’ classification models. *Mathematical Thinking and Learning*, **2**(1/2), 51-74.

Lehrer, R., & Schauble, L. (2006). Cultivating model-based reasoning in science education. In R. K. Sawyer (ed.) *The Cambridge Handbook of the Learning Sciences* (pp.79-96). St. Louis, Cambridge University Press.

Lesh, R., Cramer, K., Doerr, H., Post, T., & Zawojewksi, J. (2003). Model development sequences. In R. Lesh & H.M. Doerr (eds.) *Beyond Constructivism: A Models and Modeling Perspective on Mathematics Problem Solving, Learning and Teaching*. Lawrence Erlbaum, Hillsdale, NJ.

Lesh, R., & Harel, G. (2003). Problem solving, modeling, and local conceptual development. *Mathematical Thinking and Learning: An International Journal*, **5**(2/3), 157-190.

Lesh, R., & Zawojewski, J. (2007). Problem solving and modeling. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 763-800). Greenwich, CT: Information Age Publishing.

Loveless, T, Fennel, F., Williams, V., Ball, D., & Banfield, M. (2008). Chapter 9: Report of the Subcommittee on the National Survey of Algebra I Teachers. In *Foundations for Success: Report of the National Mathematics Advisory Panel*.

Retrieved 14 October 2010 from http://www2.ed.gov/about/bdscomm/list/mathpanel/report/nsat.pdf.
Mark, M., & Koedinger, K. (1999). Strategic support of algebraic expression writing. In Kay, J. (Ed.) *Proceedings of the Seventh International Conference on User Modeling*, (pp. 149-158). SpringerWein, New York.
Masingila, J., Davidenko, S., & Prus-Wisniowska, E. (1996). Mathematics learning and practice in and out of school: A framework for connecting these experiences. *Educational Studies in Mathematics*, **31**, 175-200.
Merriam, S., & Associates. (2002*). Qualitative Research in Practice: Examples for Discussion and Analysis*. San Francisco: Jossey-Bass.
Moll, L., & Gonzalez, N. (1997). Teachers as social scientists: Learning about culture from household research. In P. M. Hall (Ed.), Race, *ethnicity, and multiculturalism: Policy and practice* (pp. 89-114). New York: Garland.
Morgan, P., & Ritter, S. (2002). An experimental study of the effects of Cognitive Tutor® Algebra I on student knowledge and attitude. Retrieved July 2009 from http://www.carnegielearning.com/web_docs/morgan_ritter_2002.pdf.
Moschkovich, J., Schoenfeld, A., & Arcavi, A. (1993). Aspects of understanding: On multiple perspectives and representations of linear relations and connections among them. In T. A. Romberg, E. Fennema and T. P. Carpenter (Eds.) *Integrating Research on the Graphical Representation of Functions* (pp. 69-100). Hillsdale, NJ: Erlbaum.
Moses, R. (2010). Constitutional property v. constitutional people. In T. Perry, R. Moses, J. Wynne, E. Cortes, & L. Delphit (eds.) *Creating a Grassroots Movement to Transform Public Schools: Quality Education as a Constitutional Right*. Boston: Beacon Press.
Moses, R., & Cobb, C. (2001). *Radical Equations: Math Literacy and Civil Rights.* Boston: Beacon Press.
Nasir, M., Roseberry, A., Warren, B., & Lee, C. (2006). Learning as a cultural product: achieving equity through diversity. In R. K. Sawyer (ed.) *The Cambridge Handbook of the Learning Sciences* (pp.489-504). St. Louis, Cambridge University Press.
Nathan, M., Kintsch, W., & Young, E. (1992). A theory of algebra-word-problem comprehension and its implications for the design of learning environments. *Cognition and Instruction*, **9**(4), 329-389.
Nathan, M., & Koedinger, K. (2000a). An investigation of teachers' beliefs of students' algebra development. *Cognition and Instruction*, **18**, 209-238.
Nathan, M., & Koedinger, K. (2000b). Teacher and researchers' beliefs about the development of algebraic understanding. *Journal for Research in Mathematics Education*, **31**(2), 168-190.
Nathan, M., Long, S., & Alibali, M. (2002). The symbol precedence view of mathematical development: A corpus analysis of the rhetorical structure of textbooks. *Discourse Processes*, **33**(1), 1-21.
Nathan, M., & Petrosino, A. (2003). Expert blind spot among pre-service teachers. *American Educational Research Journal*, **40**(4), 905-928.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA.
National Council of Teachers of Mathematics. (2006). *Curriculum focal points for prekindergarten through grade 8 mathematics: A quest for coherence*. Reston, VA.
National Council of Teachers of Mathematics. (2009). *Executive Summary: Focus in high school mathematics: Reasoning and sense-making*. Retrieved 6 July 2010 from http://www.nctm.org/standards/content.aspx?id=23749.
Neuman, Y., & Schwarz, B. (2000). Substituting one mystery for another: The role of self-explanations in solving algebra word-problems. *Learning and Instruction*, **10**, 203-220.
Palm, T. (2006). Word problems as simulations of real-world situations: A proposed framework. *For the Learning of Mathematics*, **26**(1), 42-47.
Palm, T. (2008). Impact of authenticity on sense making in word problem solving. *Educational Studies in Mathematics*, **67**, 37-58.
Palm, T. (2009). Theory of authentic task situations. In B. Greer, L. Verschaffel, W. Van Dooren, & S. Mukhopadhyay (eds.) *Word and Worlds: Modelling Verbal Descriptions of Situations*. Rotterdam, the Netherlands: Sense Publishers.
Reed, S. (2006). Does unit analysis help students construct equations? *Cognition and Instruction*, **24**(3), 341-366.
Renninger, K., Ewen, L., & Lasher, A. (2002). Individual interest as context in expository text and mathematical word problems. *Learning and Instruction*, **12**(4), 467-490.
Resnick, L. (1987). Learning in school and out. *Educational Researcher*, **16**(9), 13-20.
Reusser, K., & Stebler, R. (1997). Every word problem has a solution: The social rationality of mathematical modeling in schools. *Learning and Instruction*, **7**(4), 309-327.
Roth, W. (1996). Where is the context in contextual word problems?: Mathematical practices and products in grade 8 students' answers to story problems. *Cognition and Instruction*, **14**(4), 487-527.
Saxe, G. (1988). Candy selling and math learning. *Educational Researcher*, **17**(6), 14-21.
Schoenfeld, A. (1988). When good teaching leads to bad results: The disasters of "well taught" mathematics courses. *Educational Psychologist*, **23**(2), 1-20.
Sfard, A. (1991). On the dual nature of mathematical conceptions: Reflections on processes and objects as different sides of the same coin. *Educational Studies in Mathematics*, **22**(1), 1-36.
Sfard, A., & Linchevski, L. (1994). The gains and pitfalls of reification – The case of algebra. *Educational Studies in Mathematics*, **26**, 191-228.
Sherman, M., & Greeno, J. (2010). Conceptual understanding in beginning algebra. Poster presentation at 2010 American Education Research Association Annual Meeting, Denver, CO.
Silver, E., Smith, M., & Nelson, B. (1995). The QUASAR project: Equity concerns meet mathematics education reform in the middle school. In W. Secada, E. Fennema, & L. Adajian (Eds.), *New directions for equity in mathematics education* (pp. 9-56). Cambridge: Cambridge University Press.

Stacey, K., & MacGregor, M. (1999). Learning the algebraic method of solving problems. *Journal of Mathematical Behavior*, **18**(2), 149-167.
Stephens, A. (2003). Another look at word problems. *The Mathematics Teacher*, **96**(1), 63-66.
Sweller, J. (1994). Cognitive load theory, learning difficulty, and instructional design. *Learning and Instruction*, **4**(4), 295-312.
Swetz, F. (2009). Word problems: Footprints from the history of mathematics. In B. Greer, L. Verschaffel, W. Van Dooren, & S. Mukhopadhyay (eds.) *Word and Worlds: Modelling Verbal Descriptions of Situations*. Rotterdam, the Netherlands: Sense Publishers.
Taylor, E. (2005). Low income first and second grade students' engagement in currency exchange: The relationship to mathematical development. University of California, Berkeley. *Doctor of Philosophy in Education*: 112.
Texas Educational Agency (2006). Released TAKS tests. Retrieved June 2009 from: http://www.tea.state.tx.us/index3.aspx?id=3839&menu_id3=793
Texas Educational Agency (2009). Released TAKS tests. Retrieved June 2010 from: http://www.tea.state.tx.us/index3.aspx?id=3839&menu_id=793
Vicente, S., Orrantia, J., & Verschaffel, L. (2007). Influence of situational and conceptual rewording on word problem solving. *British Journal of Educational Psychology*, **77**, 829-848.
Xin, Z. (2009). Realistic problem solving in China. In B. Greer, L. Verschaffel, W. Van Dooren, & S. Mukhopadhyay (eds.) *Word and Worlds: Modelling Verbal Descriptions of Situations*. Rotterdam, the Netherlands: Sense Publishers.
Wyndhamn, J., & Saljo, R. (1997). Word problems and mathematical reasoning: A study of children's mastery of reference and meaning in textual realities. *Learning and Instruction*, **7**(4), 361-382.

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